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Deriving Composite Functions as Activation Function for Neural Networks or for Wavenets

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Abstract:

This study present new idea in way to more future wonderful ideas, exploiting mathematical side of specific, through several ways employing very wide resources of mathematical functions and their properties that with natural in improve features of activation function, from these studies this study which drive an activation function through compose two (and more) functions with properties to be continuous and drivable.

We can drive membership function under fuzzy logic by this as composite function on wavelet function or vice versa, and for many different functions (as future work).

Also there idea to compose wavelets from families such SLOG mother wavelets with RASP or POLYWOG wavelets, that specifically arises from NNs representation and classification problems ,which also drivable .Each one from these functions have derivative which also can be composed to get an other activation function .

اشتقاق دالة مركبة كدالة تحفيز للشبكات العصبية أو في شبكات المويجات

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الخـــلاصة

هذه الدراسة تقدم فكرة جديدة في طريق لمزيد من الأفكار المستقبلية الرائعة تستغل الجانب الرياضياتي المختص، من خلال بعض الطرق باستخدام مصادر واسعة للدوال الرياضية وخصائصها ذات طبيعة لتحسين صفات دالة التحفيز، ومن هذه الدراسات هذه الدراسة التي تشتق دالة التحفيز من خلال تركيب دالتين(و أكثر) مع صفات كونها مستمرة وقابلة للاشتقاق .

يُمْكِنُ أَنْ نَشتقَ دالة عضويةِ بواسطة المنطقِ المضبب بهذه كدالة مركّبة على دالة مويجة أَو العكس بالعكس، وللعديد مِنْ الدوال المختلفة .

أيضاً هناك فكرة لإعْداد المويجات مِنْ عوائلِ مثل دالة الأم SLOG ودالة RASP والتي يَظْهرُ بشكل مُحدّد عن تمثيل NNs وتصنيف المشاكلِ، والتي أيضاً قابلة للاشتقاق. كُلّ واحد مِنْ هذه الدوال لَها مشتقة والتي أيضاً يُمْكِنُ أَنْ تُركب للحُصُول على دالة تنشيطِ أخرى.

Mathematics Subject Classification: 42X40

1. Introduction:

Most if not all studies and works on NNs and WNNs in this moment try to improve their works and options, from these studies what award to improve structure of the important part in network which is the activation function.

In prior study there is idea to construct activation function as summation between two sigmoid functions (which odd function) or three functions as in reference [1].

Other ways award to exploit wavelet to combine with NNs and through use wavelets as activation function in what named WNNs.

A new techniques that hybrid many ways such NNs and wavelets analysis with fuzzy logic and fuzzy learning rules in scaling function as in reference[2],or exploiting logarithmic wavelet modeling as occurred in reference[3],and more studies which that not acquainted on it .

The idea to Design and development from the electronic circuits of mathematical models in order to generate activation functions employed in the Artificial Neural Networks(ANN)in reference[4],which present six models for activation functions.

The work in this paper has been based on casting an activation function in head principle through composite the functions, that aim to make activation function in form of composite to get on a best features for function and then for data that process by this function. Several ways exploit this will shown in more accuracy.

A way depend on composite two mother wavelets (and daughters) from different wavelets families, which compose SLOG by RASP wavelet families and with POLYWAG wavelet family. While the other way by compose POLYWAG by SLOG family.

Also for attempt to improve the computations these composite functions have been derived with first derivative, and to promote the work a derivative extended to parameters wavelet scale and shift.

The properties of chosen wavelet functions were helpful to forming the function which not requisite be in composed function.

2. Some Families of Wavelets

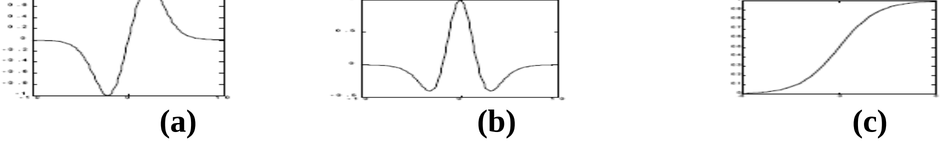
2.1 SLOG Wavelets:

This family of mother wavelets result from finite term sums of weighted and delayed logistic functions[5].A logistic function which is a type of monotonically increasing, smooth, asymptotic sigmoid (*S*-shaped) function which usually represents the threshold function at the neuron output of the neural networks models .

Sigmoid function which centered at zero $\int_{-\infty}^{+\infty} \frac{dx}{e^x+1} = 0$

The first SLOG mother wavelet exhibiting the following Superposition LOGistic sigmoids;

$$h_{s\log_1}(x) = \frac{1}{1+e^{-x+1}} - \frac{1}{1+e^{-x+3}} - \frac{1}{1+e^{-x-3}} + \frac{1}{1+e^{-x-1}} \qquad \ldots(1)$$



(a) (b) (c)

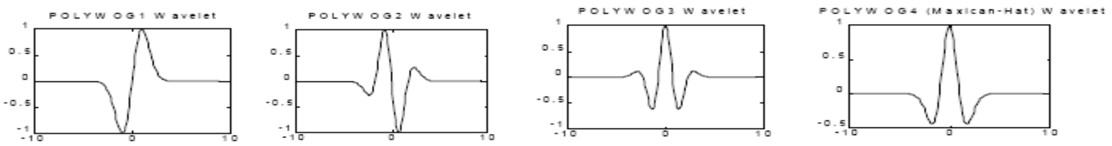
Figure (1):(a) SLOG$_1$ wavelet (b) A SLOG$_2$ wavelet (c) Sigmoid function .

Which represent a minimum-oscillation wavelet (down–up-down),its plot were normalized to have unity amplitudes .

2.2 POLWOG Wavelets:

These wavelets arise from POLYnomials WindOwed with Gaussians type of functions ,that represent all derivatives of Gaussian function $= e^{(-\frac{x^2}{2})}$,In this work a second derivative was applied which with form[5];

$$h_{polywog_2}(x) = k\ (x^3 - 3x)\ e^{-x^2/2} \quad , \quad k = 0.7246 \qquad \dots(2)$$



Figure(2):Some POLYWOG wavelets .

2.3 RASP Wavelets:

These Rational functions with Second-order Poles wavelets are real, odd functions with zero mean .The distinction among these mother wavelets is their rational form of functions being strictly proper and having simple/double poles of second order.

$RASP_1$ Wavelet with form in[5].

$$h_{Rasp_1}(x) = \frac{x}{(x^2+1)^2} \qquad \dots(3)$$

$$h_{Rasp_2}(x) = \frac{x\cos(x)}{x^2+1} \qquad \dots(4)$$

Figure (3): First and Second RASP Wavelets.

3. Compose the Activation Function:

❶ Compose of two functions as:

$$g \circ f(x) = g[f(x)]$$

for two functions f and g on x which mean circling g on f .

We can compose (or in other word circling) more than two functions. This work circling functions in (1) and (2) as putting;

$f(x) = h_{s\log_1}(x)$ and $g(x) = h_{polywog_2}(x)$, $k = 0.7246$

Compose f on g as; $g \circ f(x)$:

$$g \circ f(x) = g(f(x)) = h_{polywog_2}(h_{s\log_1}(x))$$

Represented by variable x as in form (2):

$$g(f(x)) = k[(f(x))^3 - 3f(x)].e^{[-(f(x))^2/2]} \quad , \quad k = 0.7246 \qquad \dots(5)$$

Represented by variable x as in form (1):

$$g(f(x)) = k[[\frac{1}{1+e^{-x+1}} - \frac{1}{1+e^{-x+3}} - \frac{1}{1+e^{-x-3}} + \frac{1}{1+e^{-x-1}}]^3 - 3[\frac{1}{1+e^{-x+1}} - \frac{1}{1+e^{-x+3}} - \frac{1}{1+e^{-x-3}} + \frac{1}{1+e^{-x-1}}]].e^{[-[\frac{1}{1+e^{-x+1}} - \frac{1}{1+e^{-x+3}} - \frac{1}{1+e^{-x-3}} + \frac{1}{1+e^{-x-1}}]^2/2]} \quad \ldots(6)$$

If we simplify $f(x)$ apart as in form:

$$f(x) = \frac{1+u}{e^{-2x}+u} - \frac{1+v}{e^{-2x}+v} = \frac{1+u}{w+u} - \frac{1+v}{w+v} \quad \ldots(7)$$

where $u = u(x) = 1 + e^{-x-1} + e^{-x+1}$ ….(8)

, $v = v(x) = 1 + e^{-x-3} + e^{-x+3}$ ….(9)

and $w = w(x) = e^{-2x}$ ….(10)

Or use other form ;

$$f(x) = 2 - \frac{e^{-x-3}+u}{e^{-x-1}+u} - \frac{e^{-x+3}+v}{e^{-x+1}+v} \quad \ldots(11)$$

where $u = u(x) = e^{-x-3} + e^{-2x-4} + 1$ ….(12)

and $v = v(x) = e^{-x+3} + e^{-2x+4} + 1$ ….(13)

So the composite will be as:

$$\lambda(x) = g(f(x)) = k.[[\frac{1+u}{w+u} - \frac{1+v}{w+v}]^3 - 3[\frac{1+u}{w+u} - \frac{1+v}{w+v}].e^{[-[\frac{1+u}{w+u} - \frac{1+v}{w+v}]^2/2]} \quad \ldots(14)$$

for $k = 0.7246$,and $\lambda(x)$ the implicated function.

❷ Some question requisite here what happen if the compose be inversed ?
That compose of two functions in form;

$$f \circ g(x) = f(g(x)) = h_{s\log_1}(h_{polywog_2}(x))$$

$$= \frac{1}{1+e^{-[k(x^3-3x)e^{-x^2/2}]+1}} - \frac{1}{1+e^{-[k(x^3-3x)e^{-x^2/2}]+3}} - \frac{1}{1+e^{-[k(x^3-3x)e^{-x^2/2}]-3}} + \frac{1}{1+e^{-[k(x^3-3x)e^{-x^2/2}]-1}} \quad \ldots(15)$$

,for constant value $k = 0.7246$

when facilitation form get on :

$$\vartheta(x) = f(g(x)) = \frac{1}{1+e^{-[kuv]+1}} - \frac{1}{1+e^{-[kuv]+3}} - \frac{1}{1+e^{-[kuv]-3}} + \frac{1}{1+e^{-[kuv]-1}} \quad \ldots(16)$$

where $u = u(x) = x^3 - 3x$ ….(17)

and $v = v(x)e^{-x^2/2}$ ….(18)

❸The compose of another two functions in form;

$$g \circ f(x) = g(f(x))$$

$$= h_{polywog_2}(h_{Rasp_2}(x))$$

$$= k.[[\frac{x\cos(x)}{x^2+1}]^3 - 3[\frac{x\cos(x)}{x^2+1}].e^{[-[\frac{x\cos(x)}{x^2+1}]^2/2]} \quad \ldots(19)$$

,for constant values $k = 0.7246$.

❹ The compose functions in other way in form;

$$f\circ g(x)=(h_{s\log_1}\circ h_{Rasp_2})(x)=h_{s\log_1}(h_{Rasp_2}(x))=f(g(x))$$

$$f(g(x))=\frac{1}{1+e^{-[\frac{x\cos(x)}{x^2+1}]+1}}-\frac{1}{1+e^{-[\frac{x\cos(x)}{x^2+1}]+3}}-\frac{1}{1+e^{-[\frac{x\cos(x)}{x^2+1}]-3}}+\frac{1}{1+e^{-[\frac{x\cos(x)}{x^2+1}]-1}} \qquad \ldots.(20)$$

Note: *The suggested question could executed for all contributed functions* .

❺We can compose(circling) the two functions RASP$_2$ with SLOG$_2$ wavelet with zero mean ,form(4),composed by $g(x)$ in form :

$$g(x)=h_{s\log_2}(x)=\frac{3}{1+e^{-x-1}}-\frac{3}{1+e^{-x+1}}-\frac{1}{1+e^{-x-3}}+\frac{1}{1+e^{-x+3}} \qquad \ldots.(21)$$

As ;

$$g\circ f(x)=g(f(x))=h_{Rasp_2}(h_{s\log_2}(x))$$

$$=\frac{3}{1+e^{-f(x)-1}}-\frac{3}{1+e^{-f(x)+1}}-\frac{1}{1+e^{-f(x)-3}}+\frac{1}{1+e^{-f(x)+3}} \qquad \ldots.(22)$$

If we denote the composite function for x by $\eta(x)$;

$$\eta(x)=\frac{3}{1+e^{-(\frac{x\cos(x)}{x^2+1}+1)}}-\frac{3}{1+e^{1-\frac{x\cos(x)}{x^2+1}}}-\frac{1}{1+e^{-(\frac{x\cos(x)}{x^2+1}+3)}}+\frac{1}{1+e^{3-\frac{x\cos(x)}{x^2+1}}} \qquad \ldots.(23)$$

Represented as finite terms sums series of cosines and exponential terms functions, which has parts from a sigmoid functions forms.

4. Some Properties of Composted Activation Function

Properties of functions like even $\vartheta(x)=\vartheta(-x)$, and odd function $-\vartheta(x)=\vartheta(-x)$.

For (17)and(18),function;

$$u(-x)=-(x^3-3x)=-u(x)$$

and for $v(x)=e^{-x^2/2}=v(-x)$

The features of these functions apart on the composite function, in time u is odd function, v is not ,but v is even function and u is not even function.

Composite function;

$$\vartheta(-x)=\frac{1}{1+e^{-[k((-x)^3-3(-x))e^{-(-x)^2/2}]+1}}-\frac{1}{1+e^{-[k((-x)^3-3(-x))e^{-(-x)^2/2}]+3}}-\frac{1}{1+e^{-[k((-x)^3-3(-x))e^{-(-x)^2/2}]-3}}$$

$$+\frac{1}{1+e^{-[k((-x)^3-3(-x))e^{-(-x)^2/2}]-1}}$$

$$\vartheta(-x)=\frac{1}{1+e^{kuv+1}}-\frac{1}{1+e^{kuv+3}}-\frac{1}{1+e^{kuv-3}}+\frac{1}{1+e^{kuv-1}} \qquad \ldots.(24)$$

Or $\vartheta(-x)=\frac{e^{-kuv-1}}{e^{-kuv-1}+1}-\frac{e^{-kuv-3}}{e^{-kuv-3}+1}-\frac{e^{-kuv+3}}{e^{-kuv+3}+1}+\frac{e^{-kuv+1}}{e^{-kuv+1}+1} \qquad \ldots.(25)$

When communicate the terms of function to coincide with essential function in form (16);

$$=\frac{1}{e^{-kuv+1}+1}(e^{-kuv+1})-\frac{1}{e^{-kuv+3}+1}(e^{-kuv+3})-\frac{1}{e^{-kuv-3}+1}(e^{-kuv-3})+\frac{1}{e^{-kuv-1}+1}(e^{-kuv-1})$$

It is clear that this is equivalent to essential, but with set from additional parentheses are $(e^{-kuv+1}),(e^{-kuv+3}),(e^{-kuv-3})$ and (e^{-kuv-1}), respectively. Which can represent as polynomial with power of exponential?

$$=(e^{-kuv+1})(1st\ term.e^{0}+2nd\ term.e^{2}+3rd\ term.e^{-4}+4th\ term.e^{-2})$$

$$=(e^{-kuv+1})(\frac{1}{e^{-kuv+1}+1}-\frac{1}{e^{-kuv+3}+1}.e^{2}-\frac{1}{e^{-kuv-3}+1}.e^{-4}+\frac{1}{e^{-kuv-1}+1}.e^{-2})$$

The feature of function to be centered at origin (with mean zero)in [5];

$$\int_{-\infty}^{\infty}\vartheta(x)dx=\int_{-\infty}^{\infty}f(g(x))dx$$

$$=\ln(1+e^{kuv-1})-\ln(1+e^{kuv-3})-\ln(1+e^{kuv+3})+\ln(1+e^{kuv+1})$$

$$=\ln(1)-\ln(1)-\ln(1)+\ln(1)=0$$

Integration here is through depend on k,u,v as constants with respect to values of x.

The value of function at zero is zero;

$$\vartheta(x)=\frac{1}{1+e^{1}}-\frac{1}{1+e^{3}}-\frac{1}{1+e^{-3}}+\frac{1}{1+e^{-1}}=\frac{e^{-1}}{1+e^{-1}}-\frac{e^{-3}}{1+e^{-3}}-\frac{1}{1+e^{-3}}+\frac{1}{1+e^{-1}}$$

$$=\frac{e^{-1}+1}{1+e^{-1}}-\frac{e^{-3}+1}{1+e^{-3}}=0$$

at $u(0)=0, v(0)=1$

At zero $\lambda(x)$ be also zero, at $w(0)=1$, $u(0)=1+e^{-3}+e^{-4}$, $v(0)=1+e^{3}+e^{4}$.

$$\lambda(0)=g(f(0))=k.[[\frac{1+u}{1+u}-\frac{1+v}{1+v}]^{3}-3[\frac{1+u}{1+u}+\frac{1+v}{1+v}]]e^{[-[\frac{1+u}{1+u}-\frac{1+v}{1+v}]^{2}/2]}$$

$$=k.0.e^{0}=0$$

5. Derive Composite Activation Function:

Here derive form (16) with respect to x through chain rule by functions $u\ and\ v$;

$$\frac{\partial f(g(x))}{\partial x}=\frac{\partial f(g(x))}{\partial u}.\frac{du}{dx}+\frac{\partial f(g(x))}{\partial v}.\frac{dv}{dx} \qquad ....(26)$$

$$\frac{\partial f(g(x))}{\partial u}.\frac{du}{dx}=\frac{kve^{-kuv+1}}{(1+e^{-kuv+1})^{2}}-\frac{kve^{-kuv+3}}{(1+e^{-kuv+3})^{2}}-\frac{kve^{-kuv-3}}{(1+e^{-kuv-3})^{2}}+\frac{kve^{-kuv-1}}{(1+e^{-kuv-1})^{2}}$$

Here we suppose σ as, $\sigma=kuv$ and then substitute it as;

$$\frac{\partial f(g(x))}{\partial u}.\frac{du}{dx}=kv[\frac{e^{-\sigma+1}}{(1+e^{-\sigma+1})^{2}}-\frac{e^{-\sigma+3}}{(1+e^{-\sigma+3})^{2}}-\frac{e^{-\sigma-3}}{(1+e^{-\sigma-3})^{2}}+\frac{e^{-\sigma-1}}{(1+e^{-\sigma-1})^{2}}].\frac{du}{dx} \qquad ....(27)$$

,with $\frac{du}{dx}=3(x^{2}-1)$

,and ;

$$\frac{\partial f(g(x))}{\partial v}.\frac{dv}{dx}=ku[\frac{e^{-\sigma+1}}{(1+e^{-\sigma+1})^{2}}-\frac{e^{-\sigma+3}}{(1+e^{-\sigma+3})^{2}}-\frac{e^{-\sigma-3}}{(1+e^{-\sigma-3})^{2}}+\frac{e^{-\sigma-1}}{(1+e^{-\sigma-1})^{2}}].\frac{\partial v}{\partial x} \qquad ....(28)$$

,with $\frac{dv}{dx}=-xe^{-x^{2}/2}=-xv$, for $k=0.7246$

By compile terms in (26)it will be;

$$\frac{\partial f(g(x))}{\partial x}=kv[3(x^2-1).[\frac{e^{-\sigma+1}}{(1+e^{-\sigma+1})^2}-\frac{e^{-\sigma+3}}{(1+e^{-\sigma+3})^2}-\frac{e^{-\sigma-3}}{(1+e^{-\sigma-3})^2}+\frac{e^{-\sigma-1}}{(1+e^{-\sigma-1})^2}] \quad \ldots(29)$$
$$-xu.[\frac{e^{-\sigma+1}}{(1+e^{-\sigma+1})^2}-\frac{e^{-\sigma+3}}{(1+e^{-\sigma+3})^2}-\frac{e^{-\sigma-3}}{(1+e^{-\sigma-3})^2}+\frac{e^{-\sigma-1}}{(1+e^{-\sigma-1})^2}]]$$

It is cleared here the activation function constructed from a polynomial of sigmoids and exponential functions .

6. Driving the Composted Function with Respect to Shifting and Scaling:

Since each hidden node has window in time frequency plane the parameters that control on shape of wavelet and its values which are shift and scale a_n and b_n .

$$h_{s\log_1}(x)=\frac{1}{1+e^{-\tau+1}}-\frac{1}{1+e^{-\tau+3}}-\frac{1}{1+e^{-\tau-3}}+\frac{1}{1+e^{-\tau-1}} \quad \ldots(30)$$

where $\tau=\frac{x-b_n}{a_n}$, n=1,….,N number of windowing wavelets

$$\frac{dh_{s\log_1}(x)}{d\tau}=\frac{-(-e^{-\tau+1})}{(1+e^{-\tau+1})^2}-\frac{-(-e^{-\tau+3})}{(1+e^{-\tau+3})^2}-\frac{-(-e^{-\tau-3})}{(1+e^{-\tau-3})^2}+\frac{-(-e^{-\tau-1})}{(1+e^{-\tau-1})^2}$$
$$=[\frac{e^{-\tau+1}}{(1+e^{-\tau+1})^2}-\frac{e^{-\tau+3}}{(1+e^{-\tau+3})^2}-\frac{e^{-\tau-3}}{(1+e^{-\tau-3})^2}+\frac{e^{-\tau-1}}{(1+e^{-\tau-1})^2}]$$

$$\frac{\partial\tau}{\partial b}=\frac{x-b}{a}=\frac{x}{a}-\frac{b}{a}\Rightarrow=-\frac{1}{a}$$

when driving by b_n

$$\Rightarrow\frac{\partial h_{s\log_1}(x)}{\partial\tau}.\frac{\partial\tau}{\partial b}=\frac{-1}{a}[\frac{e^{-\tau+1}}{(1+e^{-\tau+1})^2}-\frac{e^{-\tau+3}}{(1+e^{-\tau+3})^2}-\frac{e^{-\tau-3}}{(1+e^{-\tau-3})^2}+\frac{e^{-\tau-1}}{(1+e^{-\tau-1})^2}]$$

A composite function;

$$\frac{dg(f(x))}{dx}=k.[3([\frac{1}{1+e^{-x+1}}-\frac{1}{1+e^{-x+3}}-\frac{1}{1+e^{-x-3}}+\frac{1}{1+e^{-x-1}}]^3)$$
$$[\frac{-e^{-x+1}}{(1+e^{-x+1})^2}-\frac{e^{-x+3}}{(1+e^{-x+3})^2}-\frac{e^{-x-3}}{(1+e^{-x-3})^2}+\frac{e^{-x-1}}{(1+e^{-x-1})^2}]-3[\frac{1}{1+e^{-x+1}}$$
$$-\frac{1}{1+e^{-x+3}}-\frac{1}{1+e^{-x-3}}+\frac{1}{1+e^{-x-1}}]^2\ .e^{[-[\frac{1}{1+e^{-x+1}}-\frac{1}{1+e^{-x+3}}-\frac{1}{1+e^{-x-3}}+\frac{1}{1+e^{-x-1}}]^2/2]}$$

$$\frac{dg(f(x))}{dx}=\frac{\partial g(f(x))}{\partial u}.\frac{du}{dx}+\frac{\partial g(f(x))}{\partial v}.\frac{dv}{dx}+\frac{\partial g(f(x))}{\partial w}.\frac{dw}{dx} \quad \ldots(31)$$

$$\frac{\partial g(f(x))}{\partial u}.\frac{du}{dx}=k.[[3(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2(\frac{(w+u)-(1+u)}{(w+u)^2})$$
$$-3[(\frac{1+u}{w+u}-\frac{1+v}{w+v}).(-(\frac{(w+u)-(1+u)}{(w+u)^2}).(\frac{1+u}{w+u}-\frac{1+v}{w+v})).e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2/2]}$$
$$+e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2/2]}.(\frac{(w+u)-(1+u)}{(w+u)^2})]\ ].[-e^{-x-3}-e^{-2x-4}]$$

$$= -3k.\Big[\ (\frac{1+u}{w+u}-\frac{1+v}{w+v})^2(\frac{(w-1)}{(w+u)^2}) + [(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2$$

$$.(\frac{w-1}{(w+u)^2}).e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\big/2\ ]}] + e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\big/2\ ]}(\frac{(w-1)}{(w+u)^2})\ \Big].[u-1]$$

$$= -3k.\Big[\ (\frac{(w-1)}{(w+u)^2})[(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2 - (\frac{1+u}{w+u}-\frac{1+v}{w+v})^2.e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\big/2\ ]}$$

$$+ e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\big/2\ ]}]\ \Big].[u-1]$$

$$\frac{\partial g(f(x))}{\partial u}.\frac{du}{dx} = -3k.[u-1].[u-1]\Big[\ (\frac{(w-1)}{(w+u)^2})(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2$$

$$-e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\big/2\ ]}[(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2 \quad +1]\ \Big] \qquad \ldots.(32)$$

$$\frac{\partial g(f(x))}{\partial v}.\frac{dv}{dx} = k.\Big[[3(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2(\frac{-(w+v)+(1+v)}{(w+v)^2})$$

$$-3[(\frac{1+u}{w+u}-\frac{1+v}{w+v})(-(\frac{-(w+v)+(1+v)}{(w+v)^2})(\frac{1+u}{w+u}-\frac{1+v}{w+v}))e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\big/2\ ]}$$

$$+e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\big/2\ ]}.(\frac{-(w+v)+(1+v)}{(w+v)^2})]\ \Big][-e^{-x+3}-e^{-2x+4}]$$

$$= 3k.\Big[\ (\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\ \frac{1-w}{(w+v)^2} + [(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2.(\frac{1-w}{(w+v)^2}).e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\big/2\ ]}$$

$$+e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\big/2\ ]}.(\frac{1-w}{(w+v)^2})]\ \Big][v-1]$$

$$= 3k[v-1]\Big[\ \frac{1-w}{(w+v)^2}[(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2 + [(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2 e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\big/2\ ]}$$

$$+e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\big/2\ ]}]]\ \Big]$$

$$\ldots.(33)$$

$$\frac{\partial g(f(x))}{\partial w}.\frac{dw}{dx} = k.\Big[[3(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2(\frac{-(1+u)}{(w+u)^2}-\frac{-(1+v)}{(w+v)^2})$$

$$-3[(\frac{1+u}{w+u}-\frac{1+v}{w+v}).(\frac{1+u}{w+u}-\frac{1+v}{w+v}).(\frac{-(1+u)}{(w+u)^2}-\frac{-(1+v)}{(w+v)^2})).e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\big/2\ ]}$$

$$+e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2\big/2\ ]}.(\frac{-(1+u)}{(w+u)^2}-\frac{-(1+v)}{(w+v)^2})]\ \Big].[-2e^{-2x}]$$

$$= 3k.\Big[[(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2(\frac{-(1+u)}{(w+u)^2}+\frac{(1+v)}{(w+v)^2})$$

$$+[(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2.(\frac{-(1+u)}{(w+u)^2}+\frac{(1+v)}{(w+v)^2})).e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2/2]}$$

$$+e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2/2]}.(\frac{-(1+u)}{(w+u)^2}+\frac{(1+v)}{(w+v)^2})]\ \Big].[-2w]$$

$$= 3k.\Big[[(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2(\frac{-(1+u)}{(w+u)^2}+\frac{(1+v)}{(w+v)^2})+(\frac{-(1+u)}{(w+u)^2}+\frac{(1+v)}{(w+v)^2})$$

$$.e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2/2]}].[(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2+1]\ \Big].[-2w]$$

$$\frac{\partial g(f(x))}{\partial w}.\frac{dw}{dx}=-6kw.\Big[[(\frac{-(1+u)}{(w+u)^2}+\frac{(1+v)}{(w+v)^2})][(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2$$

$$+[(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2+1].e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2/2]}]\ \Big] \quad \ldots(34)$$

If we suppose the function ρ to represent the form in (34) as:

$$\rho = \frac{1+u}{w+u}-\frac{1+v}{w+v} \quad \ldots(35)$$

The previous equations will be formed as;

$$\frac{\partial g(f(x))}{\partial u}.\frac{du}{dx}=-3k[u-1].\Big[\ \frac{(w-1)}{(w+u)^2}(\rho)^2[e^{[-(\rho)^2/2]}[(\rho)^2\ \ +1]\ \Big] \quad \ldots(36)$$

$$\frac{\partial g(f(x))}{\partial v}.\frac{dv}{dx}=3k[v-1].\Big[\ \frac{1-w}{(w+v)^2}[(\rho)^2+e^{[-(\rho)^2/2]}[(\rho)^2+1]]\ \Big] \quad \ldots(37)$$

$$\frac{\partial g(f(x))}{\partial w}.\frac{dw}{dx}=6kw\rho.\Big[\ [(\rho)^2+e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2/2]}.[(\rho)^2+1]]\ \Big] \quad \ldots(38)$$

by communicating the derivatives in chain in (31);

$$\frac{dg(f(x))}{dx}=3k.\Big[\ [-(u-1)]\frac{(w-1)}{(w+u)^2}[(\rho)^2-e^{[-(\rho)^2/2]}[(\rho)^2\ \ +1]$$

$$+(v-1)\frac{(w-1)}{(w+v)^2}[(\rho)^2+e^{[-(\rho)^2/2]}[(\rho)^2+1]] \quad \ldots(39)$$

$$+2w\rho[(\rho)^2+e^{[-(\frac{1+u}{w+u}-\frac{1+v}{w+v})^2/2]}.[(\rho)^2+1]]\ \Big]$$

$$\frac{dg(f(x))}{dx}=3k.\Big[\ [-(u-1)]\frac{(w-1)}{(w+u)^2}[(\rho)^2-e^{[-(\rho)^2/2]}[(\rho)^2\ \ +1]$$

$$+[(\rho)^2+e^{[-(\rho)^2/2]}[(\rho)^2+1]][(v-1)\frac{(w-1)}{(w+v)^2}+2w\rho\ \Big] \quad \ldots(40)$$

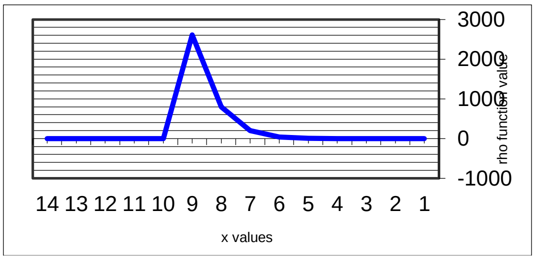


Figure (4): A form ρ from the composite form.

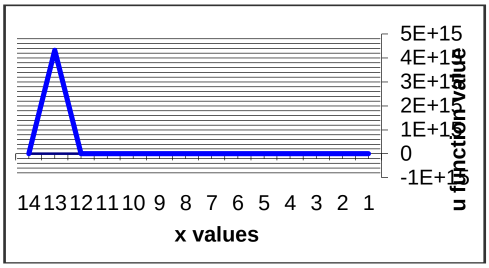


Figure (5): A function u from the composite form.

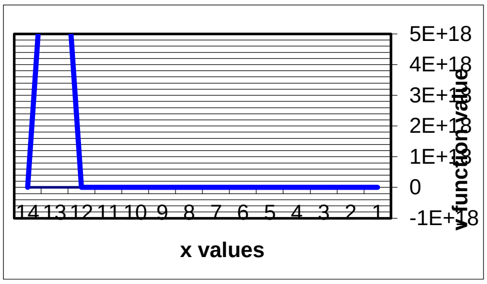


Figure (6):A function v from the composite form.

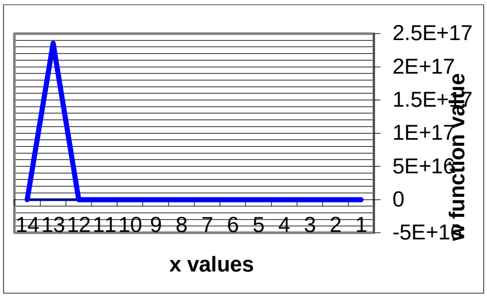


Figure (7):A function w from the composite form

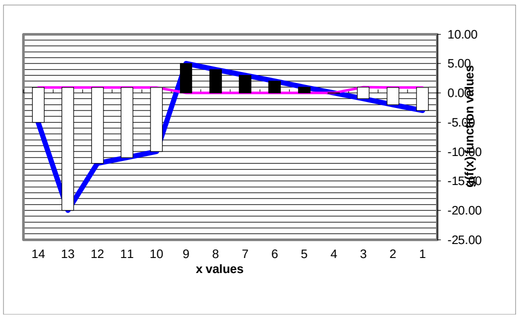


Figure (8): Graphing of $g \circ f(x)$ function

Conclusions:

The work does hard to introduce an elegant way or a form from many thoughts introduced and will be introduce. Presented thought try to restrict the computations through the form of activation function which advantage from the composite functions properties. The ensuring from the success of computations in this way as a new work in experimental calculations on NNs or WNNs (to be continued) in spite the complexity of form. The computations reflect on a features values through NNs (WNNs) in two elicit directions. In direction that drive $POLYWOG_2$,$POLYWOG_3$ on $SLOG_1$ mother wavelet and then under scaling and shifting on a variable values through a composed function to get on an activation function with special properties. In direction that compose a $RASP_2$ mother wavelet with mean zero by a $SLOG_2$ mother wavelet in once, which is near to work of Fourier transform as a series of summation of cosine functions of (shifted and scaled) variable(s) .The graph of composed function is changed from negative to positive to negative values, which clearing effective of properties of function on data.

Some computations be inaccessible, the us of activation function with form communicate many functions (exponential, trigonometric, linear, … , etc.) in sequential serial summation or/and subtraction may eliminate or null the covariance in computations, will work to extend the scope of computational intelligence and learning. For coming works will drive a composite function for membership function on/by wavelet function (or membership function) and for many functions, this feature available by association property for composite function. The range of results values from this function is clearly be accurate and little and may be intersected this by properties of functions that were composite.

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