**Connections**

**Types of connections :**

1. Bolted Connections (including rivets).
2. Welded Connections.

**1. Bolted connections.**

**1.1 Types of bolts used in connections** (depending on the strength of the bolt)

- **A.** Ordinary or common bolts (A307): Usually used in light structures subjected to light and static loads.

- **B.** High strength bolts (A325 bolts and A490 bolts): used for all types of steel structures.

On the other hand, there are three types of **high strength bolts** according to the **method of erection** and the **degree of tensioned load** applied to the bolts:

- **Snug-tight bolts:** Used in light load situations where pretension and slip-critical joints are not required.

- **Pretensioned joints:** Very high tensile stress approximately equal to 70% of the minimum tensile strength ($F_u$) of the bolts material. It is used in situations that require high load reversal (full design load is required in two directions of stress) or if the bolts are subjected to **tensions** or **combined shear** and tension.

- **Slip-critical joints.** Used in situations that require **shear** or **combined shear** and **tension** but not required for cases that involves **tension only**.

**1.2 Types of bolted connection.**

There are two basic bolted joint types used from **high strength bolts** depending on the load transfer between the bolts and the connected parts:

- **A. Slip -resistance connections:** The load is transferred between members by **friction** in the joint (the applied shearing load is less than the frictional resistance of the connection)
Table J3.1 of the AISC specification or Table (12.1), page 370 of the textbook gives the minimum bolt tension required for slip-resistance connection for both A325 AND A490 bolts.

B. **Bearing-type connections**: The load is transferred between members by bearing on the bolts (the applied shearing load is greater than the frictional resistance of the connection).

![Bearing-type connections](image)

**1.3 Sizes of bolt holes: Section J3 of the AISC Specifications**

a. **Standard size bolt holes (STD)**: can be used anywhere. Dia of holes = Dia. of bolt + 1/8

b. **Oversized holes (OVS)**: Useful in speeding up the steel erection. Gives more choices in adjustment the plumping frames during steel erection. The applied load must not exceeds the slip resistance of the connected parts so it can't be used in bearing type connections but may be used in slip-critical connections.

c. **Short -slotted holes (SSL)**: The slot must be perpendicular to the direction of load. My be used in bearing type connections.

d. **Long -slotted holes (LSL)**: May be used in only one of the connected parts in slip-critical and bearing type connection so they are primarily used when connecting to existing structures.

Table (12.2), page 375 of textbook or table J3.3 of AISC Specification provides the nominal hole dimensions for each the values of the type of hole size.

**1.4 Load transfer in bolted connections**

Case 1: When the load is assumed to pass through the center of gravity of the connectors group (axially loaded connections).
1.4.1 Failure modes

1. Single shear failure in the bolts (only in lap joint).

2. Tensile tearing failure in one of the plate through the bolt hole.

3. Bearing failure in bolts or plate between them.

4. Shearing failure out of part of the connected members.

5. Double shear failure in the bolts (only in butt joint).

Spacing and edge distances

Minimum spacing between bolts: Table J3.3 of AISC specifications provides minimum centre to centre distances between bolts for standards, oversized and slotted bolt holes to prevent bearing failure of the members between bolts. The minimum should not less than 2 2/3” of the bolts diameter.

Minimum edge distance. Table J3.4 and J3.5 of AISC specifications. (Tables 12.3 and 12.4 in the Textbook page 381)

1.5. Design strength of bearing type of connections

A. Shearing strength.

Nominal strength for single shear ($R_n$) = $F_n \times \text{area of the cross section of the bolt (} A_b = \pi D^2/4 \).

$$R_u = \phi R_n \quad \phi = 0.75 \text{ for LRFD method}$$

$F_n$ = The nominal shearing strength of the bolts and rivet in ksi and they are given in Table J3.2 in AISC Specifications (Table 12.5 in Textbook page 385).

Table J3.2 of AISC Specifications gives two values of $F_n$, for each bolts size
1. When threads are included (or **not excluded**) in the shear plane.

2. When threads are excluded from the shear plane (more strong).

If normal bolts and members sizes are used, the threads will almost always be **excluded** from the shear plane.

**B. Bearing strength (Section J3.10 of the AISC Specification).**

1. For bolts used with **standards**, **oversized** and **short slotted holes** regardless the direction of the applied load and with **long slotted holes** if the load is parallel to the slot direction.

   - If the deformation around the bolts holes are a design consideration (i.e. not accepted in design, deformations ≤0.25in).
     
     \[ R_n=1.2(l_c)(t)(F_u) \leq (2.4)(d)(t)(F_u) \]

     \[ R_u=\varphi R_n \]

     \[ \varphi=0.75 \]

     **AISC Equation J3.6a**

   - The nominal bearing strength of bolts

   \( l_c \)=the clear distance between the edge of holes and the edge of the adjacent holes or edge of the material in the direction of the force.

   \( t \)=the smaller thickness of the members (plate) bearing against the bolts.

   \( d \)= the bolt diameter.

   \( F_u \) is the minimum tensile strength of the connected parts material.

   - If the deformation around the bolts holes are not a design consideration (i.e. accepted in design, deformations >0.25in)
     
     \[ R_n=(1.5)(l_c)(t)(F_u) \leq (3.0)(d)(t)(F_u) \]

     **AISC Equation J3.6b**

2. For bolts used with **long slotted holes** and the load is perpendicular to the slot direction.

\[ R_n=(1.0)(l_c)(t)(F_u) \leq (2.0)(d)(t)(F_u) \]

**AISC Equation J3.6c**

**✓ Symbols used when referring to various types of bolts:**
A325-SC - slip-critical of fully tensioned A325 bolts.

A325-N - sung-tight or bearing A325 bolts with threads included in the shear planes.

A325-X - sung-tight or bearing A325 bolts with threads excluded in the shear planes.

Examples:

Ex1: Determine the design strength $\phi P_n$ for the bearing type connection shown below. The steel is A36 and the bolts are 7/8" A325. The holes are standard sizes and the threads are excluded from the shear plane. Assume that the deformations at bolt holes are a design consideration.

Solution:

1. Determine the design strength of the connected parts (i.e. the plates as tension members).

For A36 steel $F_y=36$ ksi and $F_u=58$ ksi

$\phi P_n$ for tensile yielding failure = $\phi (F_y \times A_g) = 0.9 \times 36 \times (1/2'' \times 12'') = 194.4$ k

$\phi P_n$ for tensile rapture failure at the effective net area = $\phi (F_u \times A_e)$

$A_e = U \times A_n = 1 \times (12 - 2(7/8 + 1/8) \times 1/2) = 5$ in$^2$

Reduction factor $U=1$ because all parts are connected

$\therefore \phi P_n = 0.75 \times 58 \times 5 = 217.5$ k

2. Determine the design strength of the bolts. (the bolts are in single shear and bearing on 1/2in plate)

Shearing strength $\phi R_n = \phi \times F_nv \times A_b \times$ Total No. of bolts in the connection.

From Table J3.2 in AISC Specifications (Table 12.5 in Textbook page 385), the value of $F_nv$ for A325 with thread excluded from the shear plane is 60 ksi.

$\therefore \phi R_n = 0.75 \times 60 \times (7/8^2 \times \pi/4) \times 4 = 108$ k

Bearing strength for the case where deformation in the standard bolts holes is a design consideration.

$\phi R_n = \phi 1.2l_t F_u \times$ Total No. of bolts in the connection $\leq 2.4 dt F_u \times$ Total No. of bolts in the connection.
or the clear distance between the edges of holes to the edge of the material in the direction of the force = 3 - (7/8 + 1/8)/2 = 3 - 1/2 = 2.5"

use \( l_c = 2" \) (the smaller value)

\[ \therefore \phi R_n = 0.75 \times 1.2 \times 2 \times 1/2 \times 58 \times 4 \leq .75 \times 2.4 \times 7/8 \times 1/2 \times 58 \times 4 \]

\[ \phi R_n = 208k > 182.7k \]

\[ \therefore \text{use } \phi R_n = 182.7k. \]

The design shearing strength is control = 108k Ans.

**Ex2:** (Problem 12.21 page 403 of textbook). For the connection shown below, \( P_u = 360k \). Determine the number of 7/8" A490 bolts required for bearing type connection, using A36 steel. Thread are not excluded from shear plane.

Solution: The bolts are in single and double shear and bearing on 7/8" and 5/8" plates.

Determine the design strength of a single bolt

**Shearing strength:** \( \phi R_n = 2 \phi F_{nv} \times A_p \), or \( \phi R_n = \phi F_{nv} \times A_p \), (single shear)

From Table J3.2 in AISC Specifications, the value of \( F_{nv} \) for A490 with thread not excluded from the shear plane is 60 ksi.

\[ \therefore \phi R_n = 2 \times 0.75 \times 60 \times [(7/8)^2 \times \pi /4] = 54k, \text{ or } \phi R_n = 0.75 \times 60 \times [(7/8)^2 \times \pi /4] = 27k \]

**Bearing strength:** Assume the deformation in the bolts holes is a design consideration (most critical situation).

\[ \therefore \phi R_n = \phi 1.2l_c t F_u \leq 2.4dt F_u \]

\( l_c \) is the clear distance between the edges of holes in the direction of the force = 3 -(7/8 + 1/8)/2 = 1/2 = 0.5"

or the clear distance between the edges of holes and the edge of the material in the direction of the force = 2.5"
\[
2-(7/8+1/8)/2=2-1/2=1.5''
\]

Use \( l_c = 1.5'' \) (the smaller value)

for the plates (5/8×16)

\[
\therefore \phi\sigma_n = 0.75 \times 1.2 \times 1.5 \times 5/8 \times 58 \leq 0.75 \times 2.4 \times 7/8 \times 5/8 \times 58
\]

\[
\phi\sigma_n = 48.9 < 57.1 \quad \text{ok}
\]

for the plates (7/8×16)

\[
\therefore \phi\sigma_n = 0.75 \times 1.2 \times 1.5 \times 7/8 \times 58 \leq 0.75 \times 2.4 \times 7/8 \times 7/8 \times 58
\]

\[
\phi\sigma_n = 68.46k < 79.8k \quad \text{ok}
\]

\[\therefore 1. \text{ No of required bolts to prevent double shear failure} = \frac{360}{54} = 6.6 \text{ say 7 bolts}\]

2. No of required bolts to prevent single shear failure = \( \frac{(2/3)360}{27} = 8.82 \text{ say 9 bolts}\)

3. No of required bolts to prevent bearing failure at the Plate (5/8×16) = \( \frac{(2/3) \times 360}{48.9} = 5 \text{ bolts}\)

4. No of required bolts to prevent bearing failure at the Plate (7/8×16) = \( \frac{360}{68.4} = 5.26 \text{ bolts}\)

\[\therefore 6 \text{ use Nine 7/8 A490 bolts}\]

**Determining the spacing of bolts in connection of cover plates to the flanges of W section**

- The factored longitudinal shear force per one inch length of the beam can be calculated from

\[
\frac{V_Q}{I} = \text{(kip/inch)}
\]

- The spacing of a pairs of bolts in the cover plate can be determined from:

\[
\text{Spacing (inches)} = \phi\sigma_n \left( R_n \text{ is the less of design shearing or bearing strength of two bolts } \right) / \text{factored shear force for inches length of the beam.}
\]

\[
\text{the above value must be checked with the AISC specification Sect E6.2 which is:}
\]

\[
\text{Maximum spacing between bolts}= \text{thinner outside plate thickness} \times 0.75 \sqrt{\frac{E}{F_y}}
\]

Also Sec. F13.3 of the AISC specification limits the area of the cover plate with 0.7 of the total flange area (including the cover plate area).
Ex3 (Problem 12.23 page 403 of textbook): For the beam shown below, what is the required spacing of 3/4 " A490 bolts with threads not excluded from shear plane) in a bearing type connections at a section where the external shear $V_D=80k$ and $V_L=160k$. Assume $l_c=1.5\text{in}$.

**Solution:**

From AISC specifications and for W24×94 (Ag=27.7 in$^2$, d=24.3in, b=9.07in, $t_f=0.875\text{in}$, $I_x=2700 \text{in}^4$).

Assume A36 steel has been used for the W-section ($F_y=36\text{ksi}$ and $F_u=58 \text{ksi}$).

check the ration of the cover plate area to the total flange area

$$\frac{12\times1}{(12\times1+9.07\times0.875)}=0.6<0.7 \quad \text{ok} \quad (\text{Sec. F13.3 AISC -Specification})$$

The factored shear force=$1.2 \times V_D+1.6 \times V_L=1.2 \times 80+1.6 \times 160=352k$

The shear force per one in. of the beam length is $\frac{VQ}{I}$

$I_{xT}=2700+2(12\times(\frac{24.3+1}{2})) = 6540.54\text{in}^4$

$Q_{max}=12\times(\frac{24.3+1}{2})=151.8\text{in}^3$

$$\therefore \frac{VQ}{I} = \frac{352 \times 151.8}{6540.5} = 8.169 \text{ k/in.}$$

Determine the design strength of two bolts (the bolts are in single shear and the bearing thickness is 1\text{in}.)

**Shearing strength** $\rho R_n = \rho \times F_{nv} \times A_b \times 2.$ (strength of two bolts)

From Table J3.2 in AISC Specifications, the value of $F_{nv}$ for A490 with thread not excluded from the shear plane is 60 ksi.

$$\therefore \rho R_n = 0.75 \times 60 \times (3/4^2 \times \pi/4) \times 2 = 39.76 \text{ k}$$

**Bearing strength** for the case where deformation in the bolts holes is a design consideration (the critical case).

$$\rho R_n = \rho \times 1.2 l_c t F_u \times 2 \leq 2.4 d t F_u \times 2.$$

$l_c = 1.5\text{"} \quad \text{assumed in the problem }$

$$\therefore \rho R_n = 0.75 \times 1.2 \times 1.5 \times 1 \times 58 \times 2 \leq .75 \times 2.4 \times 3/4 \times 1 \times 58 \times 2$$

$\rho R_n = 156.6k = 156.6k \quad \text{ok}$
∴ use $\phi R_n = 156.6 k$.

The design shearing strength is control = 39.76 k

∴ Spacing = 39.76/8.160 = 4.87 in say 4 1/2"

Check the maximum spacing between bolts = $t \times 0.75 \sqrt{\frac{E}{F_y}} = 1 \times 0.75 \sqrt{\frac{29 \times 10^3}{36}} = 21.3" > 4 \frac{1}{2}"$ ok

∴ use two 3/4 bolts at 4 1/2 "

Ans.

Ex4 (Prop. 12.37 of the textbook page 406). Determine the design tensile strength $P_u$ of the connection shown below if eight 1 in A325 bearing-type bolts (threads excluded from the shear plane) are used in each flange. Include block shear in your calculations. A572 grade 50 steel is used.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{connection_diagram}
\caption{Connection Diagram}
\end{figure}

Solution:

1. Determine the design strength of the connected parts (the plate and the W-section).

For A572 steel $F_y = 50 ksi$ and $F_u = 65 ksi$.

for the W21×101 ($A_g = 29.8 \text{ in}^2, d = 21.4 \text{ in}, b_t = 12.3 \text{ in}, t_f = 0.8 \text{ in}$).

a. For the plates

$\phi R_n$ for tensile yielding failure = $2 \times \phi (F_y \times A_g)$ = 2 × 0.9 (50 × (3/4" × 14")) = 945 k

$\phi R_n$ for tensile rupture failure at the effective net area = $2 \times \phi (F_u \times A_n)$
\[ A_n = (14'' - 2 \times (1 + 1/8)) \times 3/4 = 8.81 \text{in}^2 \]

\[ A_n \leq 0.85 \times A_g = 0.85 \times (3/4'' \times 14'') = 8.93 \text{ in}^2 \text{ (AISC Specification for the connection parts only)} \]

\[ \therefore \text{use } A_n = 8.81265 \text{in}^2 \]

\[ \therefore \phi R_n \text{ for tensile rapture failure at the effective net area= } 2 \times 0.75 \times 65 \times 8.81265 = 859 \text{kips} \]

b. For the W-section

\[ \phi P_n \text{ for tensile yielding failure= } \phi(F_y \times A_g) = 0.9 \times (50 \times 29.8) = 1341 \text{ k} \]

\[ \phi P_n \text{ for tensile rapture failure at the effective net area= } \phi(F_u \times A_n) \]

\[ A_e = U \times A_n = \]

U from table D3.1 of AISC specification for W-section with 3 or more fasteners per line with

\[ b_f = 12.3 \text{in.} < (2/3)d = 2/3 \times 21.4 = 14.27 \text{in.} \]

\[ A_c = U \times A_n = 0.85 \times (29.8 - 4 \times (1 + 1/8) \times 0.8) = 22.27 \text{in}^2 \]

\[ \therefore \phi P_n = 0.75 \times 65 \times 22.27 = 1085.67 \text{kips} \]

Check the block shear strength= \[ \phi R_n = 0.75(0.6F_u A_{nv} + U_b F_u A_{nt}) \leq 0.75(0.6F_y A_{gv} + U_b F_u A_{nt}) \]

\[ A_{gv} = (9 + 1.5) \times t = 10.5 \times 0.8 = 8.4 \text{in}^2 \]

\[ A_{nv} = (10.5 - 3.5(1 + 1/8)) \times 0.8 = 5.525 \text{in}^2 \]

\[ A_{nt} = ((12.3 - 5.5)/2 - 0.5(1 + 1/8)) \times 0.8 = 2.27 \text{in}^2 \]

\[ U_b = 1 \text{ (single row for each shear plane)} \]

\[ \therefore \phi R_n = 0.75(0.6 \times 65 \times 5.525 + 1.0 \times 65 \times 2.27) \leq 0.75(0.6 \times 50 \times 8.4 + 1.0 \times 65 \times 2.27) \]

\[ 272.26 \text{k} < 299.67 \text{ ok} \]

\[ \therefore \phi R_n = 4 \text{(four planes of shear)} \times 272.26 \text{k} = 1089 \text{k} \]

\[ \therefore \text{The design strength of the connected parts is } = 859 \text{kips} \]

2. Determine the design strength of the bolts.

Shearing strength \[ \phi R_n = \phi \times F_{nv} \times A_b \times \text{Total No. of bolts in the connection.} \text{ (single shear)} \]

From Table J3.2 in AISC Specifications (Table 12.5 in Textbook page 385), the value of \( F_{nv} \) for A325 with thread excluded from the shear plane is 60 ksi.

\[ \therefore \phi R_n = 0.75 \times 60 \times (1.2\pi/4) \times 16 = 565.58 \text{kips} \]

Bearing strength for the case where deformation in the bolts holes is a design consideration.

\[ \phi R_n = \phi 1.2l t F_u \times 16 \leq 2.4 dt F_u \times 16. \]
\( l_c \) is the clear distance between the edges of holes in the direction of the force \( = 3-(1+1/8)=3-1.125=1.875" \)

or distance between the edges of holes and edge of the material in the direction of the force \( = 1.5-(1+1/8)/2=1.5-1.125/2=0.9374" \)

use \( l_c = 0.9375" \) (the smaller value)

\[
\therefore \phi R_n = 0.75 \times 1.2 \times \frac{0.9375 \times 3/4 \times 65 \times 16 \leq 0.75 \times 2.4 \times 1 \times 3/4 \times 65 \times 16}{16} \\
\phi R_n = 658.125 \times 1404 k \text{ ok} \\
\therefore \text{The design strength of the bolts is } = 565.58 k\text{ips} \\
\therefore \text{The design strength of the connection is } = 565.58 k\text{ips} \quad \text{Ans.}
\]

2. Design strength of slip critical (slip resistance) of connections

Almost all bolted connections with standard size holes are designed as bearing type connections. Slip-critical connections should be used when the engineer think that slipping will adversely affect the serviceability of the structure such as bridges where slippage may cause excessive distortion of the structure or a reduction in strength of the connection.

The slip-critical bolts are not stressed in shear. However, the AISC specifications J3.8 provides a design strength for shear (design friction values on the faying (contact) surfaces)

The nominal slip resistance of a connection \( (R_n) \) according to AISC specifications J3.8 is:

\[
R_n = \mu D_a h_f T_a n_s \\
R_u = \phi R_n
\]

where \( \mu \) is the slip coefficient =0.3 for Class A faying surface and 0.5 for Class B faying surface

Class A ( unpainted clean, mill scale surface or surface with class A coating on blast-cleaned steel surface. Class B surfaces are unpainted blast-cleaned steel surfaces or surfaces with class B coatings.(Sec 3 Part 16.2 of AISC Manual)

\[
D_a = 1.13, \quad h_f = \text{ Factor of fillers determined as:} \quad \\
\text{ where the bolts are added to distribute loads: } h_f = 1 \\
\text{ where the bolts are not added to distribute loads: } h_f = 1 \text{ for one filler between the connected parts and } h_f = 0.85 \text{ for two or more fillers between the connected parts} \\
\]

\( T_a = \text{Minimum fastener tension (TableJ3.1 AISC specification or Table 12.1 in textbook page 370)} \)

\( n_s = \text{Number of slip panels.} \)

\( \phi = 1 \quad \text{ for standard size and short-slotted holes perpendicular to the direction of the load.} \)

\( \phi = 0.85 \quad \text{ for oversized and short-slotted holes parallel to the direction of the load.} \)

\( \phi = 0.7 \quad \text{ for long-slotted holes regardless the direction of the load.} \)
Examples

Ex 1 (Prop. 12.35 of the textbook page 406). Using the figure shown below, determine the number of 1 in. A325 bolts required for the slip -critical strength level limit state. Assume long-slotted holes in the direction of the load, Class A faying surfaces and $l_c=1.25\text{in.}$, $P_D=120\text{k}$ and $P_L=150\text{k}$. Assume the threads not excluded from the shear plane.

**Solution:**

$P_u=1.2 \times 120+1.6\times 150=384\text{ kips}$

For the slip-critical bolts

$\phi R_u = \phi \mu D_u h_f T_b n_s$

$\mu=0.3$ for **Class A** faying surface

$D_u=1.13 \ , h_f=1.0 \ , T_b=51\text{kips}$ from table J3.1 AISC specification or table 12.1 in textbook page370.

$n_s=2$ two slip panels, $\phi=0.7$ for long-slotted bolts

$\therefore R_u = 0.7\times 0.3\times 1.13\times 1 \times 51\times 2 =24\text{ kips/bolt}$ (For one bolt)

$\therefore \text{No. of bolts}=\frac{384}{24}=16\text{ bolts}$

**Check shearing and bearing strengths for the 16 bolts:**

The bolts are in double shear and bearing on 3/4" plate

**Shearing strength** $\phi R_n=2\times \phi \times F_{nv} \times A_b \times 16.$

From Table J3.2 in AISC Specifications, the value of $F_{nv}$ for A325 with thread not excluded from the shear plane is 48 ksi.

$\therefore \phi R_n=2\times 0.75\times 48\times (1^2.\pi/4) \times 16=904.6\text{k}> P_u=384\text{k}$ **ok**

**Bearing strength** for the case where deformation in the bolts holes is a design consideration.

$\phi R_n=\phi l_c t F_u \times 16 \leq 2.4 dt F_u \times 16.$

$l_c=1.25$ from the problem.

$\therefore \phi R_n=0.75 \times 1.2 \times 1.25 \times 3/4 \times 58 \times 16 \leq 0.75\times 2.4\times 3/4 \times 3/4 \times 58\times 16$

$\phi R_n=783< 940\text{k}$ **ok**

$\therefore \text{use } \phi R_n =783>P_u=384\text{k}$ **ok**

$\therefore \text{use 16 slip-critical bolts}$ **Ans.**
Ex2:
A. Select the number of ¾-in.-diameter A325 slip-critical bolts required for slip resistance with a Class A faying surface that are required to support the loads shown when the connection plates have short slots transverse to the load and one fillers are provided.

B. Repeat the example with the same loads, but assuming that the connected pieces have long-slotted holes in the direction of the load.

Solution:

A. For short slotted holes transverse to the load

\[ P_u = 1.2(17.0 \text{ kips}) + 1.6(51.0 \text{ kips}) = 102 \text{ kips} \]

\[ \phi R_n = \phi \mu D_u h_f T_b n_s \]

\[ \phi = 1 \quad \text{for short-slotted holes perpendicular to the direction of the load.} \]

\[ \mu = 0.30 \quad \text{for Class A surface,} \quad D_u = 1.13, \quad h_f = 1.0, \quad \text{factor for fillers, assuming no more than one filler} \]

\[ T_b = 28 \text{ kips, from AISC Specification Table J3.1,} \quad n_s = 2, \quad \text{number of slip planes (two plates are used).} \]

\[ \therefore \phi R_n = (1)(0.30)(1.13)(1.0)(28 \text{ kips})(2) = (19.0 \text{ kips/bolt}) \]

Required Number of Bolts,

\[ n_b = \frac{P_u}{\phi R_n} = \frac{102}{19} = 5.37 \quad \text{use 6 bolts.} \]

B. For long-slotted holes in the direction of the load

\[ \phi R_n = \phi \mu D_u h_f T_b n_s \]

\[ \phi = 0.7 \quad \text{for long-slotted holes.} \]

\[ \therefore \phi R_n = (0.7)(0.30)(1.13)(1.0)(28 \text{ kips})(2) = (13.3 \text{ kips/bolt}) \]

Required Number of Bolts,

\[ n_b = \frac{P_u}{\phi R_n} = \frac{102}{13.3} = 7.67 \quad \text{use 8 bolts.} \]

Note: To complete the design of this connection, the limit states of bolt shear, bolt bearing must be determined:

Shearing strength \( \phi R_u = \phi \times F_{nv} \times A_b \times 6 \) (single shear)
Case 2. Eccentrically loaded bolted connections

1. Bolts subjected to shear and moment (eccentric shear)

Sec. J1.7 of AISC specification presents values of computing design strength of the eccentrically loaded bolt but it does not present a method of computing the force for each individual bolt so it is left up to the designer.

Methods of analysis of eccentrically loaded connection

1. Elastic method. (very conservative method, overestimates the moment and forces in the bolts, neglects the ductility of bolts and the connected parts are assumed perfectly plastic).

2. Reduced or effective eccentricity method. (smaller eccentricity and smaller moments are used).

3. Instantaneous center of rotation method. (most realistic values but more complicated)

1. Elastic method.

Total force in each bolts= The force P divided by number of bolts in the connection + force due to the moment caused by the couple (P.e).

The distance from each bolts to the center of gravity c.g of the connection group is assumed to be d₁, d₂, d₃, etc, and the force in the bolts due to the moment (P.e) is assumed to be r₁, r₂, r₃ etc.

\[ M_{c.g.} = Pe = r_1d_1 + r_2d_2 + r_3d_3 + r_4d_4 \]

The force in each bolts is assumed to be proportional to the distance from the bolts center to the c.g of the connection.

\[ \frac{r_1}{d_1} = \frac{r_2}{d_2} = \frac{r_3}{d_3} = \frac{r_4}{d_4} \]

\[ r_1 = \frac{r_1d_1}{d_1} \quad r_2 = \frac{r_2d_2}{d_2} \quad r_3 = \frac{r_3d_3}{d_3} \quad r_4 = \frac{r_4d_4}{d_4} \]

\[ M = \frac{r_1d_1^2}{d_1} + \frac{r_2d_2^2}{d_2} + \frac{r_3d_3^2}{d_3} + \frac{r_4d_4^2}{d_4} = \frac{r_1}{d_1}(d_1^2 + d_2^2 + d_3^2 + d_4^2) \]

\[ M = \frac{r_1}{d_1}(\Sigma d^2) \]
Therefore the force on each bolts can now be written as:

\[ r_1 = \frac{M d_1}{\Sigma d^2} \quad r_2 = \frac{d_2}{d_1} r_1 = \frac{M d_2}{\Sigma d^2} \quad r_3 = \frac{M d_3}{\Sigma d^2} \quad r_4 = \frac{M d_4}{\Sigma d^2} \]

The horizontal and vertical components of the force at each bolt (H and V) can be determined from the component of the distance \( d \) (h and v) as shown from the figure below:

\[ H = \frac{r_1 v}{d_1} = \left( \frac{M d_1}{\Sigma d^2} \right) \left( \frac{v}{d_1} \right) \]

Therefore \( H = \frac{M v}{\Sigma d^2} \)

and similarly \( V = \frac{M h}{\Sigma d^2} \)

\[ P_v = V + \frac{P}{N} \quad P_h = H + \frac{P}{N} \]

\[ P_b = (P_v^2 + P_h^2)^{1/2} \]

3. **Instantaneous center of rotation method.**

This method is more realistic method of analysis of bolted connections and uses a trial and error procedure as follows:

**General trial and error procedure for the instantaneous center of rotation method.**

1. Assume a value of the location of instantaneous center of rotation \( (e') \) as shown in the figure.
2. Determine the distance from each bolts to the center of rotation \( d \) and break it to horizontal and vertical components.

3. Assume the deformation \( \Delta \) at the bolts with largest distance \( d \) is 0.34 in and determine the deformation at the other bolts from the equation.

\[ \Delta_{\text{bolt}} = \frac{d_{\text{bolt}}}{d_{\text{max}}} \times 0.34 \]

4. Determine the estimated resistance force at each bolt from the equation.

\[ R = R_{\text{ult}} \left(1 - na^{-10\Delta} \right)^{0.55} \]

Where \( R_{\text{ult}} \) is the ultimate **shearing or bearing** strength of the bolts (whichever is smaller) \( na \) is the base of the natural algorithm = 2.718.

5. Compute the vertical component of the resistance force \( R \) for each bolts \( R_v \).

6. Determine the estimated total force of the connection \( P_u \) from the equation:-
Structural Steel Design  2014-2015     Dr. Haitham A. Bady

\[ Pu.(e+e') = \sum Rd \]

\[ P_u = \frac{\sum Rd}{e' + e} \]

6. If \( P_u \neq \sum R_i \), then assume a another value of \((e')\) if \( P_u = \sum R_i \) with smaller difference then the assumed values of \((e')\) is ok and \( P_u \) is the total resistance of the bolted group.

Tables 7.7 to 7.14 of AISC specifications entitled "Coefficients C for Eccentrically Loaded Bolts Group" can be used to determine the ultimate eccentrically y loaded bolted connection strength. These tables can be used for a large percentage of the practical cases. For the cases that not included in these tables, the elastic methods can be used instead.

**Examples:**

**Ex1.** For the bearing type connection shown below with 7/8in, A325 bolts with threads excluded from the shear plane, locate the instantaneous centre of rotation of the connection using a trial and error procedure and determine the values of \( P_u \). Assume the shear strength is control.

A. Using trial and error procedure.
B. Using Table 7-7 to 7-14 of AISC Manual.

**Solution:**

**A. By trial and error procedure.**

**First trial**

1. Assume a value \((e')\) is 3 in.
2. The distance from each bolts to the center of rotation \( (d) \) and break it to horizontal and vertical components are computed and are listed in the table below.
3. The deformation at the other bolts from the equation \( \Delta_{\text{bols}} = \frac{d_{\text{bols}}}{d_{\text{bols}}} \times 0.34 \) and are listed in the tables below
4. The values of the ultimate resistance of each bolts based on shearing strength is

\[ \therefore R_{ul} = 0.75 \times F_m \times A_b \]

For 7/8", A325 bolts with threads excluded from the shear plane \( F_m = 60 \text{kips} \)

\[ \therefore R_{ul} = 0.75 \times 60 \times ((7/8)^2 \times \pi/4) = 27.06 \text{kips.} \]

The estimated resistance force at each bolt from the equation.

\[ R = 27.06(1 - 2.718^{-10A})^{0.55} \]

and these values are listed in the tables below.
5. The estimated total force of the connection $P_u$ from the equation

$$P_u = \sum \frac{R_d}{e^{'}} = \frac{455.36}{3+5} = 56.92 \text{kips} < \sum R_v = 66.6 \quad \text{N.G}$$

After several trials assume $e^{' \prime} = 2.4$ in.

Table below list the calculation

<table>
<thead>
<tr>
<th>Bolt No.</th>
<th>$h$(in.)</th>
<th>$v$(in.)</th>
<th>$d$(in.)</th>
<th>$\Delta$(in.)</th>
<th>$R$(kips)</th>
<th>$R_v$(kips)</th>
<th>$Rd$(in.-kip)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
<td>3</td>
<td>3.3541</td>
<td>0.211</td>
<td>25.15</td>
<td>11.25</td>
<td>843</td>
</tr>
<tr>
<td>2</td>
<td>4.5</td>
<td>3</td>
<td>5.4083</td>
<td>0.34</td>
<td>26.50</td>
<td>22.05</td>
<td>1433</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>3</td>
<td>3.3541</td>
<td>0.211</td>
<td>25.15</td>
<td>11.25</td>
<td>843</td>
</tr>
<tr>
<td>4</td>
<td>4.5</td>
<td>3</td>
<td>5.4083</td>
<td>0.34</td>
<td>26.50</td>
<td>22.05</td>
<td>1433</td>
</tr>
</tbody>
</table>

$$P_u = \sum \frac{R_d}{e^{'}} = \frac{418.9}{2.4+5} = 56.61_k = \sum R_v = 56.46 \quad \text{OK. \ Ans.}$$

B. Using Tables 7-7 to 7-14 of AISC Manual.

$R_{ult}$ (total) = $C \times R_{ult}$ (bolt)

$C$ = Coefficients for Eccentrically Loaded Bolts

Table 7-8 is used for this example to obtain value of $C$ (Table 7-8 used for two rows of bolts with horizontal spacing, $s=3\text{in}$ and angle =0$^\circ$)

From this table and sing vertical spacing, $s=6\text{in}$., $e_x=5\text{in}$, and No. of vertical bolts, $n=2$

The value of $C$ is 2.10

$R_{ult}$ (bolt) = 0.75×60×((7/8)^2×\pi/4)=27.06kips. (based on shear strength)

$\therefore R_{ult}$ (total) = 2.1×27=56.7k   \text{Ans.}$

Ex2. (Problems 13.9 and 13.11 pages 441 and 442 of the textbook).

A. Using the elastic method, determine the LRFD design strength of the bearing-type connection shown. The bolts are 3/4 in A325 and are in single shear and bearing on 5/8in. The holes are standards sizes and the bolt threads are excluded from the shear plane.
B. Repeat the problems using the AISC ultimate strength tables 7-7 to 7-14.

Solution:

A. Calculate the maximum force in a bolt and equalize it to the design strength of one bolt

\[ M = P_u e = P_u \times 14.75 \]

\[ \sum d = \sum h^2 + \sum v^2 = 8 \times (2.75)^2 + 4 \times (3)^2 + 4 \times (9)^2 = 420.5 \text{in} \]

The design strength of the connection depends on the resistance force of the most stressed bolts (the upright, the up left, the downright and the down left). For each of these bolts, the resistance force is:

\[ H = \frac{Mv}{\sum d^2} = \frac{M \times 9}{420.5} = \frac{(14.75P_u) \times 9}{420.5} = 0.0964P_u \]

\[ V = \frac{Mh}{\sum d^2} = \frac{M \times 2.75}{420.5} = \frac{(14.75P_u) \times 2.75}{420.5} = 0.315P_u \]

\[ \therefore R = \sqrt{(0.315P_u)^2 + (0.0964P_u + 0.125P_u)^2} = 0.385P_u \]

The design strength for a single bearing-type bolts can be bearing on 5/8in. plate determined by the following:-

Shearing strength \( \varphi R_n = 0.75 \times F_{nv} \times A_b \).

From Table J3.2 in AISC Specifications, the value of \( F_{nv} \) for A325 with thread excluded from the shear plane is 60 ksi.

\[ \therefore \varphi R_n = 0.75 \times 60 \times (3/4)^2 \times \pi/4 = 19.88 \text{kips} \quad \text{(Controls)} \]

Bearing strength for the case where deformation in the bolts holes is a design consideration.

\[ \varphi R_n = \varphi 1.2l_c t F_{u} \leq 2.4dt_{F_{u}}. \]

\( l_c \) is the clear distance between the edges of holes in the direction of the force = \( 6-(3/4+1/8)=6-0.875=5.125" \)

or \( l_c \) is the distance between the edges of holes and edge of the material in the direction of the force= \( 3-(3/4+1/8)/2=3-(0.875/2)=2.5625" \)

Use \( l_c = 2.5625" \) (the smaller value).

\[ \therefore \varphi R_n = 0.75 \times 1.2 \times 2.5625 \times 5/8 \times 58 \leq 0.75 \times 2.4 \times 3/4 \times 5/8 \times 58 \]

\[ \varphi R_n = 83.6 > 49 \text{kips} \]

\[ \therefore \text{use } \varphi R_n = 49 \text{kips} \]

\[ \therefore 0.385P_u = 19.88 \text{kips} \]

\[ P_u = 51.65 \text{kips} \quad \text{Ans.} \]
B. using tables 7-7 to 7-14.

\[ R_{ult} \text{(total)} = C \times R_{ult}(\text{bolt}) \]

C = Coefficients for Eccentrically Loaded Bolts

Table 7-9 is used for this example to obtain value of C (Table 7-9 used for two rows of bolts with horizontal spacing, s= 51/2 in and angle =0°)

From this table and sing vertical spacing, s= 6 in., and No. of vertical bolts, n=4

For \( e_x = 14 \text{ in.} \), the value of C is 3.24.
For \( e_x = 16 \text{ in.} \), the value of C is 2.9.
By interpolating , the value of C for \( e_x = 14.75 \text{ in.} \) is 3.1125

\[ R_{ult}(\text{bolt}) = 19.88 \text{ (based on shear strength)} \]

\[ \therefore R_{ult} \text{(total)} = 3.125 \times 19.88 = 61.95 \text{k} \quad \text{Ans.} \]

2. Bolts subjected to shear and tension (Bearing - type connections)

The bolts used in many number of structural steel connections are subjected to combination of shear and tension as shown in the figure.

The vertical component V is trying to shear the bolts while the horizontal component H is trying to fracture the bolts in tension.

Section J3.7 of AISC specification provides the values of the nominal modified tensile stress to include the effect of shearing force \( F_n' \) for LRFD as follows:

\[ F_n' = 1.3F_n - \frac{F_n}{\phi}f_{rv} \leq F_n \]

\[ F_{ut}' = \phi F_n' \quad \phi = 0.75 \]

\[ F_{ut}' \geq f_{rt} \]

Where

\( F_n \) is the nominal tensile stress from Table 12.5, (page 385 textbook) (AISC Manual table J3.2), ksi.
\( F_n \) is the nominal shear stress from Table 12.5, (page 385 textbook) (AISC Manual table J3.2), ksi.
\( f_{rv} \) is the required shear stress.
\( f_{rt} \) is the required tension stress.
Ex.1 (Prob. 3.15 page 442 textbook).

If the load shown in the bearing - type connection shown passes through the centre of gravity of the bolt group, how large can it be according to the LRFD method. Bolt threads are excluded from the shear planes.

Solution:

\[ V = P_u \frac{1}{\sqrt{5}} \]
\[ H = P_u \frac{2}{\sqrt{5}} \]

\[ f_{rv} = \frac{P_u \frac{1}{\sqrt{5}}}{8 \times (3/4)^2 \times \pi / 4} = 0.1265P_u \]
\[ f_{rt} = \frac{P_u \frac{2}{\sqrt{5}}}{8 \times (3/4)^2 \times \pi / 4} = 0.253P_u \]

From Table J3.2 in AISC Specifications, the values of \( F_{nt} \) and \( F_{nv} \) for A490 with thread excluded from the shear plane are 113ksi and 75ksi respectively.

\[ F_{wt}' = \phi (1.3F_{nt} - \frac{F_{nt} - F_{nv}}{\phi F_{nv}}) \leq \phi F_{nt} \]
\[ F_{wt}' = 0.75(1.3 \times 113 - \frac{113}{0.75}) \times 0.1265P_u \] (110.175 - 0.1906P_u) \leq \phi F_{wt} = 84k

\[ \therefore F_{nt}' \geq f_{rt} \]
\[ \therefore 110.175 - 0.1906P_u = 0.253P_u \]
\[ \therefore P_u = 248.7k \text{ Ans.} \]

\[ \phi F_{nv} \geq f_{rv} \]
\[ 0.75(75) = 0.1265P_u \]
\[ P_u = 444.6\text{kips} \]

Ex2: A ¾-in.-diameter ASTM A325-N bolt is subjected to a tension force of 3.5 kips due to dead load and 12 kips due to live load, and a shear force of 1.33 kips due to dead load and 4 kips due to live load. Check the combined stresses according to AISC Specification Equations J3-3a and J3-3b.

Solution:

\[ T_u = 1.2(3.50 \text{kips}) + 1.6(12.0 \text{kips}) = 23.4 \text{kips} \]
\[ V_u = 1.2(1.33 \text{kips}) + 1.6(4.00 \text{kips}) = 8.00 \text{kips} \]

From AISC Specification Table J3.2, (Table 12.5 of the textbook page 385)

\[ F_{nt} = 90 \text{ksi}, F_{nv} = 48 \text{ksi} \text{ for A325-N (with thread included in the shear plane).} \]

for a ¾-in.-diameter bolt, \( A_b = 0.442 \text{in}^2 \)

\[ f_{rv} = \frac{8}{0.442} = 18.1\text{ksi} \] \( \phi F_{nv} = 0.75(48 \text{ksi}) = 36\text{ksi} \) Ok

\[ f_{rt} = \frac{23.4}{0.442} = 53\text{ksi} \]
\[ F'_{ut} = \phi(1.3F_{ut} - \frac{F_{ut} - F_{rv}}{\phi F_{nv}}) \leq \phi F_{ut} \]
\[ F'_{ut} = 0.75(1.3 \times 90 - \frac{90}{0.75(48)}) \]
\[ F'_{ut} = 53.8125 < 0.75 \times 90 = 67.5 ksi \quad \text{Ok} \]
\[ \therefore F'_{ut} = 53.8125 ksi \geq f_{ut} = 53 ksi \quad \text{OK} \quad \text{Ans.} \]

3. Bolts subjected to shear and tension (Slip - Critical connections)

The presence of the axial tension force in the slip-critical connection causes a reduction in the design shear strength of the bolts. This reduction is accounted for by the reduction factor \( k_s \) in the AISC specification (Section J.8). The reduction factor can be determined as follows:

\[
k_s = 1 - \frac{T_u}{D_u T_b N_b} \]

Where:

\( T_u \) = the tension force due to the LRFD load.
\( D_u \) = the multiplier 1.13.
\( T_b \) = Minimum fastener tension, Table J3.1 of AISC specification (Table 12.1 of textbook page 370).
\( N_b \) = the number of bolts carrying the applied tension.

Ex1: (Prop. 13.14 page 442)
Is the slip-critical connection shown below sufficient to resist 200k load that passes through the center of gravity of the bolt group, according to the LRFD specification. Assume Class A faying surface, \( h_f = 1.00 \) and standard bolt hole sizes are used.

Solution

\( P_u = 200 k \)
\( T_u = 200 \times 4/5 = 160 K \)
\( V_u = 200 \times 3/5 = 120 K \)

For the slip-critical bolts

\( R_u = \phi \mu D_u h_f T_b N_b \)

\( \mu = 0.3 \) for Class A faying surface

\( D_u = 1.13, \ h_f = 1.0, \ T_b = 39 \) kips for 7/8” A325 bolt from table J3.1 AISC specification or table 12.1 in textbook page 370.

\( \phi = 1 \) for standard bolt hole sizes

\( \therefore \) Shear strength for one bolt (slip resistance) is \( (\phi R_u) = 1 \times 0.3 \times 1.13 \times 1 \times 39 \times 1 = 13.221 \) kips
The reduction factor is \[ k_s = 1 - \frac{T_u}{D_u T_n N_b} = 1 - \frac{160}{1.13 \times 39 \times 8} = 0.546 \]

\[ \therefore k_s \phi R_u = 0.546 \times 13.221 = 7.221 \]

Total shear strength of the connection group = 8 \times 7.221 = 57.7 \text{ kips} < V_u = 120 \text{ kips}

\[ \therefore \] Connection is insufficient \hspace{1cm} \text{Ans.} \]

**EX2**: The slip-critical bolt group shown as follows is subjected to tension and shear. Use 3/4-in.-diameter ASTM A325 slip-critical Class A bolts in standard holes. This example shows the design for bolt slip resistance only, and assumes that the beams and plates are adequate to transmit the loads. Determine if the bolts are adequate.

**Solution:**

For the slip-critical bolts:

\[ \mu = 0.30 \text{ for Class A surface} \]

\[ D_u = 1.13 \]

\[ n_b = 8, \text{ number of bolts carrying the applied tension} \]

\[ h_f = 1.0, \text{ factor for fillers, assuming no more than one filler} \]

\[ T_b = 28 \text{ kips, from AISC Specification Table J3.1 (Tabl1 12.1 in textbook page 370) for 3/4" bolt} \]

\[ n_s = 1, \text{ number of slip planes} \]

\[ P_u = 1.2(15.0 \text{ kips}) + 1.6(45.0 \text{ kips}) = 90.0 \text{ kips} \]

\[ T_u = 90 \times 4/5 = 72.0 \text{ kips} \]

\[ V_u = 90 \times 3/5 = 54.0 \text{ kips} \]

AISC Specification Table J3.2 (Tabl1 12.5 in textbook page 385) for A325 bolts, the nominal tensile strength in ksi is, \( F_{nt} = 90 \text{ ksi.} \)
for a ¾-in.-diameter bolt, \( A_b = 0.442 \text{ in}^2 \)

The available slip resistance per bolt is:

\[
\phi R_n = \phi \mu D_u h_f T_b n_s
\]

\( \phi = 1.0 \) (standard holes)

\[
\phi R_n = 1.00(0.30)(1.13)(1.0)(28 \text{ kips})(1) = 9.49 \text{ kips/bolt}
\]

Slip-critical combined tension and shear coefficient

\[
k_u = 1 - \frac{T_u}{D_u T_b N_b} = 1 - \frac{72}{1.13 \times 28 \times 8} = 0.716
\]

\[
\phi R_n = \phi R_u k_u n_b
\]

\[
= 9.49 \text{ kips/bolt}(0.716)(8 \text{ bolts})
\]

\[
= 54.4 \text{ kips} > 54.0 \text{ kips} \quad \text{Ok.} \quad \text{Ans.}
\]