**Design procedure of columns**

The design of steel columns is a trial and error process.

1. The factored load is computed.

2. The effective slenderness ratio \((kL/r)\) can be assumed to be generally between \((40\,\text{to}\,60)\) for lengths between 10ft to 15ft.

3. Based on the assumed \((kL/r)\), an estimated value of the design buckling stress can be obtained either by using AISC equations or by using AISC tables (4.22, page 4.318 of AISC Manual).

4. The estimated column area can be obtained by:

\[
A_{(estim.)} = \frac{Pu}{\phi F_{cr} \, \text{assumed}}
\]

5. Using the estimated column area, a trial column section can be selected.

6. The actual effective slenderness ratio can now be computed and the design buckling stress can be obtained for the selected section and then the column design strength \((\phi P_n)\) can be calculated by

\[
\phi P_n = \phi F_{cr} \times A_g
\]

7. The design strength \(\phi P_n\) is compared with the factored load and it must be \(\geq (Pu)\)

8. Check the local buckling for the selected section.
Example 5:

a) Using $F_y = 50$ ksi, select the lightest W14 section available for the service column loads $P_D = 130$ k and $P_L = 210$ k. Assume $k_L = 10$ ft.

b) Repeat (a) using the AISC column design tables

**Sol.**

Factored load, $P_u = 1.2P_D + 1.6P_L = 1.2(130) + 1.6(210) = 492$ k

a). Assume $\frac{kL}{r} = 50$, $ØF_{cr}$ from Table 4.22, p 381 of AISC Manual = 37.5 ksi

∴ $A_{(estimated)} = \frac{492}{37.5} = 13.12$ in$^2$

from AISC Manual, **selected W14×48** (A=14.1 in$^2$, $r_x$=5.85 in, $r_y$=1.91 in).

**Check the selected section**

$$\frac{kL}{r_{\min}} = \frac{10 \times 12}{1.91} = 62.82$$

From Table 4.22, p 381 of AISC Manual, $ØF_{cr}$ = 33.75 ksi

$ØP_n = ØF_{cr} \times A_g = 33.75 \times 14.1 = 476$ k < 492 k  Not Ok

Select W14×53 (A=15.6 in$^2$, $r_x$=5.84 in, $r_y$=1.92 in).

$$\frac{kL}{r_{\min}} = \frac{10 \times 12}{1.92} = 62.5$$, $ØF_{cr}$ = 33.85 ksi

$ØP_n = ØF_{cr} \times A_g = 33.85 \times 15.6 = 528$ k > 492 k  Ok  ∴ **Use W14×53**

b). For $k_L = 10$ ft, From Table 4.1, p.4.14 of AISC Manual,

W14×48  $ØP_n$=477 < 492 k  Not Ok

W14×53  $ØP_n$=528 k > 492 k  Ok  ∴ **Use W14×53**
Check (width/thickness) ratio for local buckling

For W14×53 (bf=8.06”, tf=.66”, k=1.25”, d=13.9”, tw=0.37”)

1. For unstiffened element (flange)

\[
\frac{b_f}{2t_f} = \frac{8.06}{2 \times 0.66} = 6.11 < 0.56 \sqrt{\frac{E}{F_y}} = 13.9 \text{ (case 3, Table B4.1) OK}
\]

2. For stiffened element (web)

\[
\frac{h}{t_w} = \frac{(13.9 - 2 \times 1.25)}{0.37} = 30.81 < 1.49 \sqrt{\frac{E}{F_y}} = 35.82 \text{ (case 10, Table B4.1) OK}
\]

∴ No local buckling occurs

• Design of columns if the effective length is different

\[
\left(\frac{kL}{r}\right)_x > \left(\frac{kL}{r}\right)_y
\]

○ The column design tables (4.1 to 4.20) of AISC Manual provides design strength of columns with respect to y-axis assuming that \(\left(\frac{kL}{r}\right)_y\) is longer than \(\left(\frac{kL}{r}\right)_x\).

○ For the case of \(\left(\frac{kL}{r}\right)_x > \left(\frac{kL}{r}\right)_y\), or if \((kL)_x > (kL)_y\), two approaches can be used:

1. Trial and error procedure

   ✓ Select a trial section assuming \(\left(\frac{kL}{r}\right) = 50\)

   ✓ Compute of \(\left(\frac{kL}{r}\right)_x\) and \(\left(\frac{kL}{r}\right)_y\)

   ✓ Determine \(\phi F_{Cr}\) based on the larger values of \(\left(\frac{kL}{r}\right)\) and multiply by \(A_g\) to determine \(\phi P_n\)

   ✓ Check the \(\phi P_n\) with factored load and select another size if necessary.
2. AISC Design tables

- A section size is to be selected from table (4.1 to 420) based on \((kL)_y\)
- Take \(\frac{r_x}{r_y}\) value from the table for that shape size and calculate \((kL)_y\) \(_{\text{Equivalent}}\) and compare with \((kL)_y\) actual.

\[
(kL)_y\_{\text{Equivalent}} = \frac{(kL)_x}{\left(\frac{r_y}{r_x}\right)}
\]

I. If \((kL)_y\) \(_{\text{Equivalent}}\) < \((kL)_y\) actual, then \((kL)_y\) \_actual controls the design and the selected shape is correct.

II. If \((kL)_y\) \(_{\text{Equivalent}}\) > \((kL)_y\) actual, then \((kL)_x\) controls the design and another section must be selected based on the value of \((kL)_y\) \(_{\text{Equivalent}}\).

**Example 6:** Select the lightest satisfactory W12 for the following conditions: \(F_y = 50\) ksi, \(P_D = 250\) k, \(P_L = 400\) k, \(K_xL_x = 26\) ft, and \(K_yL_y = 13\) ft.

a. By trial and error procedure.

b. Using LRFD tables.

**Sol.**

**a. Using trial and error:**

Factored load, \(P_u = 1.2P_D + 1.6P_L = 1.2(250) + 1.6(400) = 940\) k

a). Assume \(\frac{KL}{r} = 50\) \(\phi_F\) from Table 4.22, p 381 of AISC Manual = 37.5 ksi

\[
\therefore A_{(estimated)} = \frac{900}{37.5} = 25.067in^2
\]

from AISC Manual, selected **W12×78** (A = 25.6 in\(^2\), \(r_x = 5.38\) in, \(r_y = 3.07\) in).
Check the selected section

\[
\frac{kL}{r} x = \frac{26 \times 12}{5.38} = 57.94 \quad \text{Control}
\]

\[
\frac{kL}{r} y = \frac{13 \times 12}{3.07} = 50.81
\]

From Table 4.22, p 381 of AISC Manual, \( OF_{cr} = 35.2 \text{ ksi} \)

\[
\psi P_n = \psi F_{cr} \times A_g = 35.2 \times 25.6 = 901 \text{ k} < 940 \text{ k} \quad \text{Not Ok}
\]

**Selected larger section, say W12×96** (\( A = 28.2 \text{ in}^2, r_x = 5.44 \text{ in}, r_y = 3.09 \text{ in} \)).

Check the selected section

\[
\frac{kL}{r} x = \frac{26 \times 12}{5.44} = 57.35 \quad \text{Control}
\]

\[
\frac{kL}{r} y = \frac{13 \times 12}{3.09} = 50.48
\]

from Table 4.22, p 381 of AISC Manual, \( OF_{cr} = 35.55 \text{ ksi} \)

\[
\psi P_n = \psi F_{cr} \times A_g = 33.55 \times 28.2 = 1002.5 \text{ k} > 940 \text{ k} \quad \text{Ok}
\]

\[\therefore \text{Use W12×96}\]

b). Using LRFD tables (Table 4.1 of AISC Manual)

Based on \((KL)_x = 13 \text{ ft}\), Select \( W 12 \times 87 \) from Table 4.1, p.4.17 of AISC Manual

From same table, \( \frac{r_x}{r_y} = 1.75 \). Since \((KL)_x = 26 > (KL)_y = 13\), calculate the equivalent \((KL)_y \) from the equation:

\[
(kL)_{y, \text{Equivalent}} = \left(\frac{(kL)_x}{r_x}\right) \times \frac{26}{1.75} = 14.8 \text{ ft} > (KL)_y = 13 \text{ ft}
\]

\[\therefore \text{Use } (KL)_{y, \text{Equivalent}} \text{ to determine the section size}\]

Based on \((KL)_{y, \text{Equivalent}} = 14.8 \text{ ft}\), Select \( W 12 \times 96 \), \( \psi P_n = 994.5 \text{ k} > 940 \text{ k} \) \( \quad \text{Ok}\)

\[\text{Use W12×96}\]
**Check (width/thickness) ratio for local buckling**

For W12×96 (bf=12.2”, tf=.9”, k=1.5”, d=12.7”, tw=0.55”)

1. For unstiffened element (flange)

\[
\frac{b_f}{2t_f} = \frac{12.2}{2 \times 0.9} = 6.76 < 0.56 \sqrt{\frac{E}{F_y}} = 13.9 \quad \text{(case 3, Table B4.1)} \quad \text{OK}
\]

2. For stiffened element (web)

\[
\frac{h}{t_w} = \frac{(12.7 - 2 \times 1.5)}{0.55} = 17.7 < 1.49 \sqrt{\frac{E}{F_y}} = 35.82 \quad \text{(case 10, Table B4.1)} \quad \text{OK}
\]

∴ No local buckling occurs

• **Design of built up compression members**

Built-up compression members sections are constructed with more than one shape built-up into a single member. They include:

1. They may consist of parts in contact with each other, such as cover-plated sections.

![Cover-plated Sections](Diagram)

2. They may consist of parts in near contact with each other, such as pair of angles: These pairs of angles may be separated by a small distance from each other equal the thickness of the end connection or gusset plates between them.

![Pair of Angles](Diagram)

3. They may consist of parts that are spread well apart, such as pairs of channels:

![Spread Channels](Diagram)
For long columns, it may be suitable to use built-up sections where the parts of the columns are **widely separated** from each other to give **higher moment of inertia** such as towers.

**General Notes**

- When a pair angles are used as a compression member, they need to be **fastened together** so they will act as a unit. Welds may be used at intervals or they may be connected with bolts.

- The widely spaced parts of these types must be carefully **laced or tied** together.

- The built-up section must have sufficient connection between its parts to prevent slippage on each other and produce high moment of inertia.

- Connections are usually placed at column **ends** and column **mid-span**.

**For Example**

![Diagram of column cross section and deformed shape](image)

\[ I = \frac{2bd^3}{12} = \frac{bd^3}{6} \]

\[ P_u \]

Plates deform equal amounts

\[ I = \frac{b(2d)^3}{12} = \frac{b(8d^3)}{12} = \frac{4}{6}bd^3 \]

Plates deform equal amounts

2-32
Connections requirements for built up columns

a. Components are in contact with each other (AISC Specification (Sec. E6):

The design strength of compressive built-up members whose components are in contact with each other is the same as the usual columns with ONE exception as follows:

- If the members tends to buckle about an axis parallel to the direction of connection between the W-shape and the plates, the connections are subjected to shear force, $\frac{kL}{r}$ have to modified by AISC equations (E6-1 and E6-2) as follows:

1. For intermediate connector that are made by high strength bolts (snug-tight):

$$\left(\frac{kL}{r}\right)_m = \sqrt{\left(\frac{kL}{r}\right)_o^2 + \left(\frac{a}{r_i}\right)^2}$$

(E6-1)

2. For intermediate connector that are welded or have pre-tensioned bolts

$$\left(\frac{kL}{r}\right)_m = \sqrt{\left(\frac{kL}{r}\right)_o^2 + 0.82\frac{\alpha^2}{(1+\alpha^2)}\left(\frac{a}{r_{ib}}\right)^2}$$

(E6-2)

$kL/r_m$ is the modified slenderness ratio of the built-up compressive members.

$kL/r_o$ is the original slenderness ratio of the built-up compressive members in the buckling direction.

$a$ is the distance between connections (in the long direction).

$r_i$ is the minimum radius of gyration of the individual components.
\( r_{ib} \) is the radius of gyration of the individual components in the axis parallel to the axis of buckling (i.e. if moment about x-axis we take \( r_x \) and if moment about y-axis we take \( r_y \)).

\( \alpha \) is the separation ratio = \( \frac{h}{2r_{ib}} \) where \( h \) is the distance between centroids of the individual components perpendicular to the axis of buckling.

(AISC Specification (Sec. E7):

\[
\frac{ka}{r_i} \leq \frac{3}{4} \left( \frac{kL}{r} \right) \text{ Whole members}
\]

**Example 6**

You are to design a column for \( P_D = 750 \text{k} \) and \( P_L = 1000 \text{k} \), using \( F_y = 50 \text{ksi} \) and \( K_L = 14 \text{ft} \). A W12×120 is on hand. Design cover plates to snug-tight bolted at 6” spacing to the W section.

**Sol.**

For W12×120, (\( A = 35.3 \text{in}^2 \), d=13.1”, bf=12.3”, \( I_x = 1070 \text{in}^4 \), \( I_y = 345 \text{in}^4 \)).

\( P_u = 1.2 \times 750 + 1.6 \times 1000 = 2500 \text{k} \)

Using trial and error:

a). Assume \( \frac{kL}{r} = 50 \) \( \Phi_{cr} \) from Table 4.22, p381 of AISC Manual =37.5ksi

\[ \therefore A_{\text{estimated}} = \frac{2500}{37.5} = 66.67 \text{in}^2 \]

\( A_{\text{plates}} = 66.67 - 35.5 = 31.36 \text{in}^2 \) (Area for each plate =15.68in²)

Try Two cover plates PL1×16

**Check the selected section**

\( A = 35.3 + 2(1 \times 16) = 67.3 \text{ in}^2 \)

\[ I_x = 1070 + 2 \times 16 \times \left( \frac{13.1 + 1.0}{2} \right)^2 = 2660 \text{in}^4 \]
Calculate the modified slenderness ratio for \( x \)-axis (parallel to the connection plane):

\[
\left( \frac{kL}{r} \right)_m = \sqrt{\left( \frac{kL}{r} \right)_o^2 + \left( \frac{a}{r_i} \right)^2}
\]

(Snug-tight bolted connections are used)

\( a = 6'' \)

\( r_r = r_x = \sqrt{\frac{I_x}{A}} \) plates (minimum radius of gyration of individual member)

\[
I_x = \frac{16 \times 1^3}{12} = 1.33 \text{in}^4 \quad A = 16 \text{in}^2 \quad r_i = r_x = 0.289''
\]

\[
\frac{a}{r_i} = \frac{6}{0.289} = 20.76, \quad \left( \frac{kL}{r} \right)_x = \frac{14 \times 12}{6.29} = 26.71
\]

\[
\therefore \left( \frac{kL}{r} \right)_m = \sqrt{(26.71)^2 + (20.76)^2} = 33.83 < 42.97 \quad \text{does not control the design}
\]

Check slenderness ratio limit of the plates:

\[
\left( \frac{ka}{r_i} \right) = 20.76 \leq \frac{3}{4} \left( \frac{kL}{r} \right) = \frac{3}{4} \times 42.97 = 32.23 \quad \text{Ok}
\]

For \( \left( \frac{kL}{r} \right)_y = 42.97 \) : from Table 4.22, p 381 of AISC Manual, \( \Theta F_c = 39.31 \) ksi
\[ \Omega P_n = \Omega F_{cr} \times A_g = 39.31 \times 67.3 = 2646 \text{kN} < 2500 \text{kN} \quad \text{Ok} \]

Use \textbf{W12x120} steel section with two cover plates \textbf{PL1x16}, \( F_y = 50 \text{ksi} \)

b. Components are Not in contact with each other- Design of lacing and tie plates

(AISC Specification (Sec. E6):

1. Design of tie plates (discussed in tension members)

2. Design of lacing

\[ \text{a.} \quad \left( \frac{\text{The distance between lacing connections}}{r_{\text{min(individual member)}}} \right) \leq \frac{3}{4} \left( \frac{kL}{r} \right)_{\text{whole member}} \]

\[ \text{b.} \quad V_u \text{ (on the lacing)} = 0.02P_u \text{ (whole compression member)} \]

perpendicular to the compression member

\[ \frac{kL}{r}_{\text{lacing}} \leq 140 \quad \text{for single lacing} \]

\[ \frac{kL}{r}_{\text{lacing}} \leq 200 \quad \text{for double lacing} \]

If \( l \leq 15\text{in} \), it is ok to use single lacing made with angles.
Example 7

a) Using AISC LRFD specifications and Select a pair of 12-in standard channels for the column and load shown using Fy= 50 ksi. For connection purposes, the back-to-back distance of the channels is to be 12 in. $P_D=100k$ and $P_L=300k$.

b) Using Fy =36ksi, design bolted single lacing for the column assume that ¾” bolts are used.

Sol.

$Pu=1.2P_D+1.6P_L=1.2(100)+1.6(300)=600k$

a). Assume $\frac{kL}{r} = 50$  $\varnothing F_{cr}$ from Table 4.22, p 381 of AISC Manual =37.5ksi

$\therefore A_{(estimated)} = \frac{600}{37.5} = 16.0\text{in}^2$

$A_{(required\ for\ each\ section)} = 8\text{in}^2$

From AISC-Specification(Table 1.5, P1-34), Select C12×30: ($A=8.81\text{in}^2$, d=12”, $x' = 0.674"$, $I_x=162\text{in}^2$, $I_y=5.12\text{in}^2$)

Check the selected section

$A= 17.62\text{in}^2$

$I_x=2\times162=324\text{in}^4$  

$I_y=2\times[5.12+8.81\times(\frac{12}{2}-0.674)^2]=510\text{in}^4$

$r_x = \sqrt{\frac{I_x}{A_{total}}} = \sqrt{\frac{324}{17.63}} = 4.29"$  

$r_y = \sqrt{\frac{I_y}{A_{total}}} = \sqrt{\frac{510}{17.63}} = 5.38"$

$\therefore r_{\min} = r_y = 5.38\"$, k=1.0 (from table C2.2.1, pinned ends)

$\left(\frac{kL}{r}\right)_{max} = \frac{1\times20\times12}{4.29} = 55.94$
From Table 4.22, p 381 of AISC Manual, \( \Omega F_{cr} = 35.82 \text{ ksi} \) (By interpolation)

\[ \Omega P_n = \Omega F_{cr} \times A_g = 35.82 \times 17.62 = 631\text{k} > 600\text{k} \quad \text{Ok} \]

**Check (width/thickness) ratio for local buckling**

For **C12×30** (d=12”, bf=3.17”, tf=.501”, TW=0.510 k=1.25”)

1. For unstiffened element (flange)

\[
\frac{b_f}{t_f} = \frac{3.17}{0.501} = 6.33 < 0.56 \sqrt{\frac{E}{F_y}} = 13.9 \quad \text{(case 3, Table B4.1)} \quad \text{OK}
\]

2. For stiffened element (web)

\[
\frac{h}{t_w} = \frac{(12-2\times1.25)}{0.510} = 19.12 < 1.49 \sqrt{\frac{E}{F_y}} = 35.82 \quad \text{(case 14 , Table B4.1)} \quad \text{OK}
\]

∴ No local buckling occurs

Use **2C12 × 30** steel sections

b). Design the lacing

- **The distance between lines of bolts**

  \[ = l = 8.5'' < 15'' \] it is ok to use single lacing

- **Check the slenderness limit for individual member**

  Assume \( 60^\circ \) inclination with longitudinal axis of the compression member:

∴ **Length of channel between the lacing connections** = \( 2 \times 8.5 \times \tan 30 = 9.81'' \)

- The slenderness limits

\[
\left( \frac{\text{The distance between lacing connections}}{r_{\text{min(individual-member)}}} \right) \leq \frac{3}{4} \left( \frac{KL}{r} \right)_{\text{whole-member}}
\]
\[
\left(\frac{9.81}{0.761}\right) = 12.9 \leq \frac{3}{4}(55.94) = 41.96 \quad \text{Ok}
\]

- **Calculation the force on the lacing channel**

\[Vu = 0.02 \times P_u = 0.02 \times 631 = 12.62 \text{k} \implies \frac{1}{2} V_u = 6.31 \text{ (shear force on each plane of lacing)}\]

\[\therefore \text{Compression force on the lacing section} = \frac{1}{2} \frac{V_u}{\cos(30)} = 7.28 \text{ k}\]

- **Calculation the dimensions of the lacing flat bar**

\[r = \frac{1}{\sqrt{A}} = \sqrt{\frac{(b \times t^3)/12}{b \times t}} = 0.289t\]

Assume \(\frac{L}{r} = \text{minimum value of the single lacing} = 140\)

\[r = \frac{L}{140} = \frac{9.81}{140} = 0.07\]

\[\therefore r = 0.289t = 0.07 \quad t = 0.242'' \quad \text{Try (\(\frac{1}{4}\)’’ flat bar} \quad \frac{L}{r} = \frac{9.81}{0.289 \times 1/4} = 136 < 140 \quad \text{ok}\]

From table 4.22 of AISC Manual \(\varnothing F_{cr} = 12.2 \text{ ksi for} \frac{L}{r} = 136\)

\[A_{\text{required}} = \frac{7.28}{12.2} = 0.597 \text{in}^2 \quad \therefore \text{(\(\frac{1}{4}\)’’ x 2.39’’) is needed}\]

- **Total length of the lacing** = length of lacing between connections + 2 x minimum edge distance

\[= 9.81 + 2 \times \frac{1}{4} = 12.3'' \text{ say 14’’} \quad \text{(Min. edge distance for \(\frac{3}{4}\) bolt is 1 \(\frac{1}{4}\) from table J3.4 of AISC manual)}\]

\[\therefore \text{Use (\(\frac{1}{4}\)’’ x 2 \(\frac{3}{8}\) x 1’’ 2’’) flat bar}\]
Column used in steel frames

- The effective length of a column is a property of whole structure of which the column is a part.

- The most common method of obtaining the effective length is to use the alignment charts for column braced or un-braced against sidesway, using Tables C.C2.3 and C.C2.4 of AISC manual part 2 or table 7.2 (a and b) Page 201 of textbook.

- To determine k factor by alignment charts, the following steps are followed:

  1. Compute (G) value at each end of the column ($G_A$ and $G_B$) from the following equations:

     $$G = \frac{\sum \left( \frac{4EI}{L} \right)_{forColumns}}{\sum \left( \frac{4EI}{L} \right)_{forBeams}} = \frac{\sum \left( \frac{I}{L} \right)_{Columns}}{\sum \left( \frac{I}{L} \right)_{Beams}}$$

  2. Select the appropriate alignment chart [sidesway prevented (braced) and sidesway un-prevented (un-braced)] and draw a straight line on the chart between $G_A$ and $G_B$ values and read k value.

  3. For the column bases the following G values can be used

     a) For pinned column, $G=10$

     b) For rigid connection to footing (fixed), $G=1.0$
The most important assumptions based on which the alignment charts are made are:

1. The members are elastic.
2. All columns buckle simultaneously.
3. For **braced frames**, $\theta_1=\theta_2$ for each beam, (single curvature)
4. For **un-braced frames**, $\theta_1=-\theta_2$ for each beam, (double curvature)

- For frames at which assumptions 3 and 4 are not satisfy, used the following multiplier can be used for (G) values such as the following frames:

  - Depending on the direction of ends moments $M_1$ and $M_2$

  - \( \frac{I}{L} \) \times 1.5 for sidesway prevented (braced- or pinned)
  - \( \frac{I}{L} \) \times 0.5 for sidesway not prevented (un-braced or rolled)

  - Depending on the direction of ends moments $M_1$ and $M_2$

  - \( \frac{I}{L} \) \times 2.0 for sidesway prevented (braced)
  - \( \frac{I}{L} \) \times 0.67 for sidesway not prevented (un-braced or guided support)
If the column behaviour is inelastic (assumption 1 is not valid), the column stiffness factor will be smaller $\frac{E \varepsilon I}{L}$, and the G value will be smaller and k values will be smaller (i.e. high column resistance).

- For an inelastic column situation

$$G_{inelastic} = G_{elastic} \times \text{stiffness reduction factor (SRF)}, \quad \tau_a$$

Table 4.21 of AISC Manual, page 4.317, or table 7.2 page 211 of textbook provides values of stiffness reduction factor $\tau_a$ for different values of $\frac{P_u}{A}$.

- To design columns to consider inelastic columns behaviour in frames, the following steps should be followed:
  1. Calculate $P_u$, assume $\frac{kL}{r}$ and select a trial section size.
  2. Calculate $\frac{P_u}{A}$ and select $\tau_a$ from table 4.21 in Manual or table 7.2 in textbook.
  3. Compute values of $G_{elastic}$ and multiply by $\tau_a$ and find $k$ value.
  4. Compute the $\frac{kL}{r}$ actual and the find $\Delta F_{cr}$ then determine $P_u = \Delta F_{cr} \times A_g$ and compare it with $P_u$.

**Example 8:**

a) Select a W section for column AB of the un-braced frame shown in Fig. below assuming that we have elastic behaviour, PD=450k and PL=700k, $F_y=50$ksi

b) Repeat part (b) if inelastic column behaviour is considered.

Assume that columns above and below column AB are the same size as AB
Sol.

Pu=1.2P_D+1.6P_L=1.2(450)+1.6(700)=1660k

a). Assume \( \frac{kL}{r} = 50 \) \( \Phi_{fc} \) from Table 4.22, p 381 of AISC Manual =37.5ksi

\[ A_{(estimated)} = \frac{1660}{37.5} = 44.267 \text{in}^2 \]

From AISC-Specification (Table 1.1), the following selects are available

- **W14×176**: (A=51.8in²)
- **W12×170**: (A=50.07in²) we have to use a larger section to account for the increasing in k values
- **W14×159**: (A=46.7in²)
- **W12×152**: (A=44.7in²)
- **W12×136**: (A=39.9in²)

.\( .\) For W12×170 : (A=50in², \( I_x=1650\text{in}^4 \), \( r_x=5.74\text{in} \)) we assume that the columns are bent about x- direction

\[ G_A = \frac{\sum (\frac{I}{L})_{columns}}{\sum (\frac{I}{L})_{beams}} = \frac{2\left(\frac{1650}{12 \times 12}\right)}{2\left(\frac{800}{30 \times 12}\right)} = 5.16 \]

\[ G_B = G_A = 5.16 \text{ (Same columns and beams dimensions)} \]

From alignment chart (Table C.C2.4) (un-braced frame) \( k=2.3 \)

\[ \frac{kL}{r} = \frac{2.3 \times 12 \times 12}{5.74} = 57.7 \]

From Table 4.22, p 381 of AISC Manual \( \Phi_{fc}=35.29\text{ksi} \)

\( \Phi_{Pn} = \Phi_{Fc} \times A_g = 35.29 \times 50 = 1764 > 1660k \) Ok

b). Inelastic design

Because the inelastic resistance of steel column is greater than the elastic resistance, we select a lighter section than the elastic behavior section.

Try W12×136: (A=39.9in², \( I_x=1240\text{in}^4 \), \( r_x=5.58'' \))
\[ \frac{Pu}{A} = \frac{1660}{39.9} = 41.6 \text{in}^2 \]
\[ \tau_a = 0.1974 \text{ from table 4.21 in Manual or table 7.2 in textbook.} \]
\[ G_A = \frac{2 \left( \frac{1240}{12 \times 12} \right) \times \tau_a}{\frac{800}{30 \times 12}} = 0.765 \]

From alignment chart (Table C.C2.4) (un-braced frame) \( k = 1.27 \)
\[ \therefore \left( \frac{kL}{r} \right) = \frac{1.27 \times 12 \times 12}{5.58} = 32.77 \]

From Table 4.22, p 381 of AISC Manual \( \phi_{F_c} = 41.7 \text{ksi} \)

\[ \phi_{Pn} = \phi_{F_c} \times A_g = 41.7 \times 39.9 = 1663 > 1660 \text{k} \quad \text{Ok} \]

**Example 9:**
We desire to select a W14 section for column CD in the figure shown for which \( P_D = 300 \text{k} \) and \( P_L = 600 \text{k} \), and \( F_y = 50 \text{ksi} \), Only in-plane behaviour is considered. Furthermore, assume that the columns immediately above and below CD are approximately the same size as CD, and also that all the other assumptions on which the alignment charts are met.

a) Assume elastic behaviour.

b) Assume inelastic behaviour.

**Sol.**
\[ Pu = 1.2P_D + 1.6P_L = 1.2(300) + 1.6(600) = 1320 \text{k} \]
\[ \frac{kL}{r} = 50 \quad \phi_{F_{cr}} \text{ from Table 4.22, p 381 of AISC Manual} = 37.5 \text{ksi} \]
\[ \therefore A_{(estimated)} = \frac{1320}{37.5} = 35.2 \text{in}^2 \]

From AISC-Specification (Table 1.1), the following sections are available
W14×120: (A=35.3in², Ix=1380in⁴, rx=6.24in)

Because beam dimensions are large; we expect k value will be small.

From AISC Manual and for \textbf{W30×99, Ix=3999}

\[
G_A = \frac{\sum(I/L)_{columns}}{\sum(I/L)_{beams}} = \frac{2(1380)}{2(3999)} = 0.83
\]

\[G_B = G_A = 0.83 \quad \text{Same columns and beams dimensions}\]

From alignment chart (Table C.C2.4) (isendway uninhibited (un-braced frame))

\[k = 1.25\]

\[\therefore \frac{(kL)}{r} = \frac{1.25 \times 15 \times 12}{6.24} = 36.055\]

From Table 4.22, p 381 of AISC Manual \(\Phi F_c=40.9\)ksi

\(\Phi P_n = \Phi F_{cr} \times A_g = 40.9 \times 35.3 = 1443.77k > Pu=1330k\) Ok

b). Inelastic design

Because the inelastic resistance of steel column is greater than the elastic resistance, we select a lighter section than the elastic behavior section.

Try W12×109: (A=32in², Ix=1240in⁴, rx=6.22")

\[\frac{Pu}{A} = \frac{1330}{32} = 41.25\text{in}^2\quad \tau_a = 0.217\text{ by interpolation from table 4.21 in Manual or table 7.2 in textbook.}\]

\[G_{Top} = \frac{2(1240)}{15 \times 12} \times 0.217 = 0.162\]

\[G_{Top} = G_{Bottom} = 0.162\]

From alignment chart (Table C.C2.4) (un-braced frame) \[k = 1.1\]
From Table 4.22, p 381 of AISC Manual $\Omega F_c = 41.75\text{ksi}$

$\Omega P_n = \Omega F_c \times A_g = 41.75 \times 32.0 = 1336\text{k} > 1330\text{k} \quad \text{Ok}$

**Design of Base Plate**

- Design procedure for column base plate.

1. Determine the required area of the base plate according to the design bearing strength of concrete beneath the base plate. The following equation can be used:

$$A_1 = \frac{P_u \phi_c \left(0.85 f'_c\right)}{0.6} \sqrt{\frac{A_2}{A_1}}$$

$$\phi_c = 0.6, \quad \sqrt{\frac{A_2}{A_1}} \leq 2.0 \quad \text{(in this equation only)}$$

**Notes:** Base plate area must be larger than the area of the column section ($b \times d$)

2. Determine the base plate dimensions ($N \times B$) from the following equations:

$$\Delta = 0.5(0.95d - 0.8b_f)$$

$$N = \sqrt{A_1} + \Delta$$

$$B = \frac{A_1}{N}$$

$d$ and $b_f$ are the depth and flange width of the column cross section.
Notes: values of \( B \) and \( N \) are selected to nearest 1” or 2” so that values of \( m \) and \( n \) shown in the figure are roughly equal and the cantilever moment are equals too.

3. Determine the base plate thickness from the following equation:

\[
t_{req} = l \times \sqrt{\frac{2Pu}{0.9F_y \times B \times N}}
\]

\( l \) is the largest values of \( m, n \) and \( n' \)

\( m \) and \( n \) are shown in the figure which are the cantilever dimensions from the base edge in \( x \) and \( y \) axes respectively.

\[
n' = \sqrt{\frac{db_f}{4}}
\]

Example 10: Design a base plate of A36 steel for a W 12×65 column with \( F_y = 50 \)ksi that supports loads of \( PD = 200 \)k and \( PL = 300 \)k. The concrete has a compressive strength of \( fc' = 3 \)ksi and the footing has dimensions of 9ft×9ft.

Sol.

\( Pu = 1.2 \times 200 + 1.6 \times 300 = 720 \)k

For W 12×65 (\( d = 12.1 \)”, \( b_f = 12.0 \)”)

\[
\sqrt{A_2} = 2 (the \ maximum \ value)
\]

\[
\therefore A_1 = \frac{Pu}{\phi_c (0.85fc')} = \frac{720}{0.6(0.85 \times 3) \times 2} = 235.3 \text{in}^2
\]

\( 235.2 \text{in}^2 > d \times b_f = 145.2 \text{in}^2 \quad \text{ok} \)

\( \Delta = 0.5(0.95d - 0.8b_f) = 0.5(0.95 \times 12.1 - 0.8 \times 12) = 0.9475" \)
\[ N = \sqrt{A_1} + \Delta = \sqrt{235.3} + 0.9475 = 16.3" \text{ say 16"} \]
\[ B = \frac{A_1}{N} = \frac{235.3}{16} = 14.7" \]

Use 16”×16” square base plate.

- **Check the bearing strength of the concrete**

\[ \Phi P_n = \Phi c (0.85 f'_c) A_1 \sqrt{A_1} = 0.6 \times (0.85 \times 3) \times (16)^2 \times 2 = 783.4k > 720 \text{ ok} \]

- **Determine the thickness of the base plate**

\[ m = (16 - 0.95 \times d)/2 = (16 - 0.95 \times 12.1)/2 = 2.25" \]
\[ n = (16 - 0.8 \times b_f)/2 = (16 - 0.8 \times 12)/2 = 3.2" \]
\[ n' = \frac{\sqrt{12.1 \times 12}}{4} = 3.01" \]

\[ l = \text{the largest value} = 3.2" \]
\[ t_{req} = 3.2 \times \sqrt{\frac{2 \times 720}{0.9 \times 36 \times 16 \times 16}} = 1.38" \text{ say 1 ½"} \]

**Use PL 1½”×16”×1’ 4” A36 base plate**

**Example 11** (Prob 7.15 page 235 of textbook): Design a base plate of A36 steel for a W 14×120 column with Fy=50ksi that supports loads 0f PD=150k and PL=350k. The footing size is 10ft by 10 ft and \( f'_c = 3 \text{ksi} \) and

**Sol.**

\[ P_u = 1.2 \times 150 + 1.6 \times 350 = 740k \]

For W 12×65 (d=14.5”, bf=14.7”)
\[ \sqrt{A_1} \]
Assume \( \sqrt{A_1} = 2 \text{ (the maximum value)} \)
\[ A_1 = \frac{P_t}{\phi (0.85 f'c)} \left( \frac{A_2}{A_1} \right)^{1/2} = \frac{740}{0.6(0.85 \times 3) \times 2} = 241.83 \text{in}^2 \]

\[ 241.83 \text{in}^2 > d \times b = 213 \text{in}^2 \quad \text{ok} \]

\[ \Delta = 0.5(0.95d - 0.8b) = 0.5(0.95 \times 14.51 - 0.8 \times 14.7) = 1.01" \]

\[ N = \sqrt{A_1 + \Delta} = \sqrt{241.83 + 1.01} = 16.55" \text{ say } 17" \]

\[ B = \frac{A_1}{N} = \frac{241.83}{17} = 14.22" \text{ say } 15" \]

Use 17"×15” rectangular base plate.

- **Check the bearing strength of the concrete**

\[ \bar{P}_n = \phi c (0.85 f'c') A_1 \left( \frac{A_2}{A_1} \right)^{1/2} = 0.6 \times (0.85 \times 3) \times (15 \times 17) \times 2 = 780.3k > 740 \text{ ok} \]

- **Determine the thickness of the base plate**

\[ m = \frac{16 - 0.95d}{2} = \frac{17 - 0.95 \times 14.5}{2} = 1.6125" \]

\[ n = \frac{16 - 0.8bf}{2} = \frac{15 - 0.8 \times 14.7}{2} = 1.2525" \]

\[ n' = \frac{\sqrt{14.5 \times 14.7}}{4} = 3.65" \]

\[ l = \text{the largest value } = 3.65" \]

\[ t_{req} = 3.65 \times \sqrt{\frac{2 \times 740}{0.9 \times 36 \times 15 \times 17}} = 1.544" \text{ say } 1\frac{1}{8}" \]

Use PL 1\frac{1}{8}"×15"×1' 5" A36 base plate