Design of an RLC Compensator for a Synchronous Motor: Torque Ripple Improvement

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Abstract

AC-drive systems based Permanent Magnet Synchronous Motor (PMSM) are widely utilized in industrial applications due to many advantages including excellent efficiency, best reliability, and low-effective cost compared with DC-drive systems. Design and implementation of an RLC compensator for a PMSM-drive system is demonstrated in this paper. The main aims of using an RLC compensator are decreasing of Total-Harmonics-Distortion (THD) of the input line current and Ripple Factor (RF) of the electromagnetic torque over a wide range variation in rotor speed and load. The entire transfer function of the proposed system is derived for stability verification under the change of rotor speeds. The proposed PMSM-drive system is implemented with and without RLC compensator for two cases; which are at a fixed load and a step-change in load. Good performance is achieved using the proposed RLC compensator; in terms of THD and RF are 0.77% and 3.13%, respectively.

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1. Introduction

AC-drive systems are essential for industrial and home utilizations due to (i) satisfying reliability, (ii) excellent efficiency and (ii) low maintenance cost compared to DC drives and AC drives based induction motors. In spite of these numerous advantages, PMSM-drive systems have, minor drawback related to increasing ripple in electromagnetic torque. The variations in rotor speed are increasing ripple in torque, which should be decreased by using adequate techniques [1]. In [2], a comparison between three control techniques has been demonstrated that related to chattering reduction for a speed controller. In [3], a hybrid space-vector based on Pulse Width Modulation (PWM) technique has been represented for minimizing the ripple in electromagnetic torque for a PMSM-drive system based on different switching topologies. Basically, the ripple in electromagnetic torque of a PMSM-drive system causes mechanical stresses. In that paper, an accurate speed controller has been implemented to minimize mechanical vibrations and noise [4]. For electric vehicles, high torque with a wide speed range is required. Few disadvantages in PMSM-drive systems, such as cogging reluctance torque which causes ripples in torque [5].Typical Field-Oriented Control (FOC) for PMSM-drive systems based on dynamic mathematical equations, so the variations of parameters (e.g. electromagnetic interface, distorted flux and harmonics in the line currents), lead to low performance. In addition these drawbacks cause: oscillations in speed, ripples in torque, distortions in line currents consequently increasing THD [6].

The cause of increasing ripple in torque is due to the trapezoidal wave form of the flux around the air-gap and variable mutual reluctance due to the shape of stator slots [7]. The market of PMSM-drive systems is growing...
because of low cost and high reliability. These drive systems have conventional utilizations for fast dynamic position drives and machine manipulation shaft drives, due to their torque-to-losses ratios, the effects of ripple in torque that decay the machine lifespan and position path especially at high-speeds, the noises and losses are increased [8]. In [9], a dead time compensation strategy has been proposed to mitigate the motor torque ripple for an 80 kW PMSM. In that paper, RF has been reduced from 30.3% to 22.3%. In [5], the vibration caused in a driver was analyzed for a PMSM. It was obtained that in low speeds, the drive system is sensitive to torque ripple which was 10 N.m without any compensator system. In [10], a developed method based on a dead-beat region between conduction and natural commutation zones has been presented for the torque-ripple reduction in a PMSM-drive system. The input signals of the controller were back-emf and the change of the back-emf during the commutation period. In [11], a developed integrated dual-output converter including switch selector has been proposed for reducing ripples in torque during a commutation period. Good performance has been achieved compared with a conventional inverter. In addition, the ripples in torque during the commutation period have been improved in [12] by optimizing the current-loop controller of a PMSM-drive system. It was illustrated in [13] that the optimizing of the reluctance torque to a minimum value is not adequate to get low ripples in torque. In [14], an adaptive controller based on self-commissioning was proposed for a PMSM-drive system according to ripples reduction in torque. A torque predictive controller based on the improved voltage vector controller has been experimentally implemented in [15] for torque-ripple minimizing. Also, in [16], the design of a PMSM has been improved based on rotor permanent-magnets skewing angle for decreasing the ripples in torque. Improved field-reconstruction controller has been represented in [17] for minimizing the ripples in torque for a PMSM-drive system which causes many problems; such as vibration during operation and increasing harmonics in back-emf.

In this paper, an RLC compensator is designed for a PMSM-drive system based synchronous motor to achieve minimum ripples in electromagnetic torque by improving THD of input line current. The design is based on optimally matching between RLC compensator and a PMSM under the variations of rotor speeds and mechanical torque. The entire transfer function of the drive system is derived for stability analysis. The simulation results show the high performance using RLC compensator in a wide range of rotor speeds (500-1500) rpm. The scheme of the PMSM-drive system studied in this paper is shown in Fig.1, including a PMSM, a three-phase inverter, and an RLC compensator.

**Figure 1. The proposed PMSM-drive system**

2. Modelling of a PMSM and speed controller

2.1. PMSM Model

The dynamic equations of a PMSM (surface mounted type) in the three-phase abc domain can be represented as below [18].

\[ \nu_{ab}(t) = R_i i_a(t) + \frac{dq_{ab}(t)}{dt} \]

\[ \nu_{bd}(t) = R_i i_b(t) + \frac{dq_{bd}(t)}{dt} \]

\[ \nu_{cd}(t) = R_i i_c(t) + \frac{dq_{cd}(t)}{dt} \]

where \( R_i \) is the stator winding resistance; \( i_a(t), i_b(t) \) and \( i_c(t) \) are the abc stator currents, respectively; \( \nu_{ab}(t), \nu_{bd}(t) \) and \( \nu_{cd}(t) \) are the abc stator voltages, respectively and \( \psi_a(t), \psi_b(t) \) and \( \psi_c(t) \) are the abc stator fluxes, respectively. However, the stator fluxes are the resultant of the flux produces from stator windings and the flux of rotor permanent magnets. Thus, stator fluxes can be presented in the rotating-reference frame as follows:

\[ [\nu_{abc}(t)] = \begin{bmatrix} L_{ab} & M & M \\ M & L_{bc} & M \\ M & M & L_{cb} \end{bmatrix} [i_{abc}(t)] + \lambda_m \begin{bmatrix} \sin(\theta_i) \\ \sin(\theta_i - 2\pi / 3) \\ \sin(\theta_i + 2\pi / 3) \end{bmatrix} \]

\[ T_a(t) - T_m = J_m \frac{d\omega(t)}{dt} + B_m \omega(t) \]

where \( L_{ab} \) and \( M \) are the self-inductance and the mutual-inductance of the stator windings, respectively; \( \lambda_m \) is a permanent magnetic flux of magnets on the rotor; \( \theta_i \) is the electrical position of the rotor; \( J_m \) is the rotor moment of inertia; \( B_m \) is the friction coefficient; \( T_m \) is electromagnetic torque; \( T_i \) is load torque; \( \omega \) is the electrical angular velocity; \( \omega(t) \) is rotor speed. The purpose of using a three-phase abc model is to optimally design an RLC compensator for depressing THD of input line current as minimum as possible and consequently reducing the torque-ripple.

To the design an RLC compensator for an AC-drive system, the phasor diagram of a PMSM can be represented as an RLE series circuit in Fig. 2. The total stator impedance \( Z \) includes \( R_s \) and \( L_s \) (where \( L_s \) is the equivalent stator inductance); rotor back-EMF \( (E) \) is equivalent to \( (\lambda_m \omega) \), which varies with rotor speeds.

**Figure 2. An equivalent circuit of a PMSM per phase**

2.2. Speed Controller in dq Frame

In this paper, the standard vector control based on dq frame is considered for adjusting the rotor speed of the PMSM in order to investigate the relationship between the change of rotor speed and the ripples in torque. The basic dq equations are illustrated as follows:[6,19]

\[ \nu_d = R_i i_d + L_d \frac{di_d}{dt} - \omega_s L_q i_q \]

\[ \nu_q = R_i i_q + L_q \frac{di_q}{dt} + \omega_s L_d i_d + \omega \lambda_m \]

\[ T_e = \frac{3p}{2} \left[ \lambda_m i_q - (L_d - L_q) i_d i_q \right] \]

\[ T_o = 1.5 \rho \lambda_m i_q \]

where the parameters and variables of above equations can be represented as follows; \( p \) is the number of pole-pairs; \( L_d \) and \( L_q \) are the d-axis and q-axis stator inductances, respectively; \( i_d \) and \( i_q \) are the d-axis and q-axis domain stator currents, respectively. Fig. 3 illustrates the block diagram of the speed controller used with the proposed RLC compensator. It is
based vector speed control strategy. The measured three-phase line currents are compared with the three-phase reference line currents, then adjusted by a PI controller for generating pulses to the inverter via a logical controller. The outer loop of speed controller (i.e. the difference between the reference speed \( \omega_0^* \) and the measured rotor speed \( \omega_0 \)) is designed to generate the reference current controller. It can be seen that the reference \( \text{d-axis} \) current is considered zero in order to reduce the copper losses. Also, the electromagnetic torque will include only the effective torque \( (3p/2L_d i_q) \) and the reluctance torque \( (3p/2(L_d - L_q) i_q) \) will be zero.

Figure 3. Block diagram of the speed controller

3. Design and Analysis of an RLC Compensator

3.1. Design of RLC compensator Procedures

The main objective of this paper is designing an RLC compensator for a PMSM-drive system. In [20] and [21], an RLC compensator has been proposed for decreasing THD of input current and RF of electromagnetic torque. The RLC compensator provides complexities in selecting its parameters that ensure stable operation of a PMSM-drive system under the variations of rotor speed and load. Basically, an RLC compensator is essential for mitigation a range of frequencies (from the lower cut-off frequency \( \omega_0 \) to the upper cut-off frequency \( \omega_2 \)). The bandwidth \( B \) of the RLC compensator must be related to the high switching frequency of the inverter in order to compensate the input reactive power delivered to PMSM. The function of the RLC compensator is to reduce the high order harmonics of the input current at the switching frequency of the voltage source inverter (VSI). Fig. 4 shows the equivalent single line diagram of the proposed RLC compensator considering the higher terms of harmonics in the line current of PMSM. Where \( V_{inv} \) is the voltage supplied by a VSI, \( i_{inv} \) is the input line current including harmonics; \( LC \) is inductor and capacitor in series; \( i_{inv} \) is the current n-harmonic components after compensation; \( i_{motor} \) is the motor line current with reduced n-harmonic components and \( V_{in} \) phase voltage of the motor. It is worth noting that design of the proposed RLC compensator is based on selecting its resonant frequency less than 50% of the switching frequency of the VSI in order to avoid resonance drawbacks.[22]. The optimal values of \( R, L, \) and \( C \) of the compensator are calculated to obtain the best performance [23, 24]. It is worth noting that the reactive power \( Q \) is calculated at full-load.

\[
R_f = B \frac{L_f}{\omega_0 \omega_0 C_f} \tag{10}
\]

\[
L_f = \frac{1}{\omega_0 \omega_0 C_f} \tag{11}
\]

\[
C_f = \frac{Q}{\omega_0 \omega_0 V^2} \tag{12}
\]

3.2. Analysis of the proposed RLC compensator

The equivalent circuit of RLC compensator connected with a PMSM is shown in Fig. 5. The above equivalent circuit can be simplified using the star-delta conversion method; the new delta impedances are shown in Eqs.(13-15).

\[
Z_{13} = \frac{Z_{Z2} + Z_{R_2} + Z_{Z1}}{Z_2} \tag{13}
\]

\[
Z_{12} = \frac{Z_{Z2} + Z_{R_2} + Z_{Z1}}{R_i} \tag{14}
\]

\[
Z_{23} = \frac{Z_{Z2} + Z_{R_2} + Z_{Z1}}{Z_{i}} \tag{15}
\]

where \( Z_i = sL_i + \frac{1}{sC_i} \) and \( Z_2 = R_i + sL_i \)

Figure 4. An equivalent single line diagram of the proposed RLC compensator

Figure 5. An equivalent circuit of a PMSM with an RLC compensator per phase

The entire transfer function of a PMSM connected with an RLC compensator is obtained as follows:

\[
E = \frac{V_{in}}{Z_{12} + Z_{23}} \tag{16}
\]

\[
TF = \frac{E}{V_{in}} = \frac{Z_{23}}{Z_{12} + Z_{23}} \tag{17}
\]

by substituting the amount of \( Z_i \) and \( Z_2 \) in Eq.(15) and simplifying the new form to be as follows:

\[
Z_{23} = n_1 + n_2 + n_3 \tag{18}
\]

where \( n_1 = R_i (sL_i + \frac{1}{sC_i}) \), \( n_2 = R_i (sL_i + sL_i) \) and \( n_3 = (sL_i + \frac{1}{sC_i}) (R_i + sL_i) \)
\[ Z_{23} = \frac{n_1 + n_2 + n_2}{sC_L} \] and \[ Z_{13} = \frac{n_1 + n_2 + n_2}{sC_f R_f} \]
then:
\[ TF|_b = \frac{n_1 + n_2 + n_2}{sC_L + sC_f R_f} \]
\[ TF|_b = \frac{n_1 + n_2 + n_2 + n_1 + n_2 + n_2}{sC_f R_f + sC_L} \]
\[ TF|_{\text{load}} = \frac{\omega_m}{V_{\text{ref}}} = \frac{C_f R_f}{sC_f L_f \lambda_m + (C_f R_f \lambda_m + C_f R_f \lambda_m)} \] (19)

For stability analysis, the entire transfer function (i.e. the ratio of the rotor speed to the inverter voltage) is determining in Eq. (20) using the parameters listed in Table 1.

\[ TF_{\text{load}} = \frac{18.2}{5.4 \times 10^{-3} s + 3.4} \] (20)

The root locus analysis is used to verify the stability of PMSM-Driver system supplied by the inverter via RLC compensator under the variation of rotor speed. Fig. 6 shows the root locus of Eq. (20), which includes a single pole, has the characteristics of (Gain=0; Pole=-6.23e-5; Damping=1 and Overshot=0%). It is observed that the system is stable for the optimum values of RLC compensator for 500 rpm, 1000 rpm, and 1500 rpm.

![Figure 6. Root locus of the entire transfer function](image)

### 4. Simulation Study

In this Section, the implementation of PMSM-drive system is demonstrated using the simulation parameters listed in Table 1, where \( f_{sw} \) is the switching frequency of the inverter. The block diagram of the Simulink model is illustrated in Fig. 7. For performance verification, two scenarios (with and without RLC compensator) are studied which are: (i) at a fixed load and (ii) a step-change in load; under the variation of rotor speed (500-1500) rpm.

#### Table 1. Parameters of the proposed drive-system

<table>
<thead>
<tr>
<th>PMSM (Surface mounted type) Parameters[25]</th>
<th>( R_s )</th>
<th>Stator winding resistance</th>
<th>2.87 ( \Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_s )</td>
<td>Stator inductances in rotary reference frame</td>
<td>8.5 mH</td>
<td></td>
</tr>
<tr>
<td>( \lambda_m )</td>
<td>Permanent magnetic flux</td>
<td>0.175 mWb</td>
<td></td>
</tr>
<tr>
<td>( J )</td>
<td>Rotor moment of inertia</td>
<td>( 8 \times 10^{-7} \text{kg.m}^2 )</td>
<td></td>
</tr>
<tr>
<td>( p )</td>
<td>Number of pole-pairs</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RLC compensator Design Parameters</th>
<th>( f_{sw} )</th>
<th>Switching frequency</th>
<th>25 kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_r )</td>
<td>Resonance frequency</td>
<td>12.5 kHz</td>
<td></td>
</tr>
<tr>
<td>( f_{o1} )</td>
<td>Lower cut-off frequency</td>
<td>( 1.571x10^3 \text{rad/sec} )</td>
<td></td>
</tr>
<tr>
<td>( f_{o2} )</td>
<td>Upper cut-off frequency</td>
<td>( 3.666x10^3 \text{rad/sec} )</td>
<td></td>
</tr>
<tr>
<td>( Q )</td>
<td>Quality factor</td>
<td>1.1456</td>
<td></td>
</tr>
<tr>
<td>( R_c )</td>
<td>Compensator resistance</td>
<td>50 ( \Omega )</td>
<td></td>
</tr>
<tr>
<td>( L_c )</td>
<td>Compensator inductor</td>
<td>250 ( \mu \text{H} )</td>
<td></td>
</tr>
<tr>
<td>( C_t )</td>
<td>Compensator capacitor</td>
<td>725 ( \mu \text{F} )</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 7. block diagram of the simulation model](image)

### 4.1. Results at a Fixed Load

At full-load, a simulation test and its findings are illustrated in Figures 8-10. In Figures 8-(c,9-c, and 10-c), the electromagnetic torque at steady-state shows the rigid performance of the proposed RLC compensator under the change of rotor speeds (500 rpm, 1000 rpm, and 1500 rpm). It is observed that the steady-state peak-peak torque-ripple in case of without RLC compensator are (0.150 N.m), (0.154 N.m) and (1.162 N.m) for 500 rpm, 1000 rpm, and 1500 rpm respectively. While in the case of with the proposed RLC compensator are (0.045 N.m), (0.044 N.m) and (0.040 N.m) for 500 rpm, 1000 rpm, and 1500 rpm respectively. Figures 8-(a,9-a, and 10-a) show line current waveforms without RLC compensator at 500 rpm, 1000 rpm, and 1500 rpm respectively. It is observed that distortion increased with increasing the rotor speeds and the THD are always above 5% as listed in Table 2. While Figures 8-(b,9-b, and 10-b) show line current waveforms with the proposed RLC compensator at 500 rpm, 1000 rpm, and 1500 rpm respectively. It is clear that the waveforms are sinusoidal for all rotor speeds and the THD are always less than 1% as presented in Table 2.
Figure 8. Simulation results at a fixed load for rotor speed 500rpm

(a) Line currents without RLC compensator
(b) Line currents with RLC compensator
(c) Electromagnetic torque comparison

Figure 9. Simulation results at a fixed load for rotor speed 1000rpm

(a) Line currents without RLC compensator
(b) Line currents with RLC compensator
(c) Electromagnetic torque comparison
Table 2. Performance results for various rotor speeds

<table>
<thead>
<tr>
<th>Rotor Speed(rpm)</th>
<th>500</th>
<th>750</th>
<th>1000</th>
<th>1250</th>
<th>1500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque Ripple</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without Compensator</td>
<td>13.34%</td>
<td>13.5%</td>
<td>14.67%</td>
<td>41.33%</td>
<td>83.33%</td>
</tr>
<tr>
<td>With Compensator</td>
<td>3.13%</td>
<td>3.20%</td>
<td>3.20%</td>
<td>3.06%</td>
<td>3.13%</td>
</tr>
<tr>
<td>Power Ripple</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without Compensator</td>
<td>11.71%</td>
<td>12.2%</td>
<td>15.91%</td>
<td>50.92%</td>
<td>82.76%</td>
</tr>
<tr>
<td>With Compensator</td>
<td>2.92%</td>
<td>3.26%</td>
<td>3.31%</td>
<td>3.11%</td>
<td>3.09%</td>
</tr>
<tr>
<td>Current THD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without Compensator</td>
<td>7.23%</td>
<td>7.29%</td>
<td>7.81%</td>
<td>10.78%</td>
<td>18.93%</td>
</tr>
<tr>
<td>With Compensator</td>
<td>0.77%</td>
<td>0.78%</td>
<td>0.89%</td>
<td>0.67%</td>
<td>0.77%</td>
</tr>
</tbody>
</table>

Fig. 11 shows the frequency spectrum vs. THD of the input line current at various rotor speeds with and without RLC compensator. It can be seen that THDs with RLC compensator are improved in comparison with the previous case study.

4.2. Results at a Step Change in Load

To verify the sensitivity of the proposed RLC compensator under the load change from full-load to half-load during 0.1 sec. Simulation results are presented in Fig. 12. It can be observed from Table 3 that the percentage increase in torque ripple with respect to full-load are 1.46%, 1.67%, 1.13%, 0.33% and 1.6% at 500 rpm, 750 rpm, 1000 rpm, 1250 rpm and 1500 rpm, respectively. These percentages are always less than 2%.

Table 3. Simulation results at a step change in load

<table>
<thead>
<tr>
<th>Speed</th>
<th>Ripple Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full-Load</td>
</tr>
<tr>
<td>500</td>
<td>3.07%</td>
</tr>
<tr>
<td>750</td>
<td>3.06%</td>
</tr>
<tr>
<td>1000</td>
<td>3.13%</td>
</tr>
<tr>
<td>1250</td>
<td>3.80%</td>
</tr>
<tr>
<td>1500</td>
<td>2.93%</td>
</tr>
</tbody>
</table>
5. Conclusion

This paper developed an RLC compensator for a PMSM-drive system to improve its performance, i.e. minimizing the THD of the line current and ripples in electromagnetic torque. In addition, this paper reviewed the control methods and drive topologies for torque-ripple reduction. The existing control methods for torque-ripple reduction were based on improving the standard vector control using intelligent techniques, which are so sensitive with rotor speed variations. In typical methods, RLC filters have been implemented, which were suitable for fixed-speed drive systems. In this research, an RLC compensator is improved for torque-ripple reduction under a wide range of rotor speeds (500-1500) rpm. The proposed RLC compensator is optimally designed and the stability analysis is verified using the root locus method. The line current of the PMSM with the proposed RLC compensator includes less-oscillations than without RLC compensator, hence reduction of THD is observed consequently the torque-ripple is improved. Results have shown that in case of without RLC compensator, the maximum values of THD of line current and RF of torque are 18.93% and 83.33%, respectively. While in the case with the proposed RLC compensator the maximum values of THD and RF are 0.89% and 3.20%, respectively.

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