Comprehensive Review of Natural Convection Heat Transfer in Annulus Complex Enclosures

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ABSTRACT

The natural convection heat transfer has many applications in engineering like solar collectors, cooling of electronic equipment, and geothermal engineering. The present work demonstrates the recent publications in the last ten years in this specific subject for a body located in complex shapes like rhombic, wavy, trapezoidal, elliptical, and Parallelogrammic enclosure. Many parameters like Ra, Nu, number of undulations, the position of the inner body had been addressed and discussed to draw the main conclusions and recommendations. It is worthy to mention that a wavy enclosure has been investigated less than the other simple enclosure shapes due to its complexity. Besides that entropy generation should be included in future studies in complex shapes of enclosure as this will helps the researchers to extend their studies. The inner bodies inside trapezoidal, parallelogrammic enclosure are very limited, and more investigation should be done. The review concluded for the different shapes of enclosure with the tables that illustrate the major finding of each study. Finally, the governing equations of the natural convection of enclosure filled with pure fluid, porous medium, and nanofluid had been addressed.

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1. Simple Enclosures

The natural convection heat transfer in simple enclosure shapes as illustrated in Fig. 1 had been studied numerically by various researchers due to its importance in energy related applications. Some of these studies were [1-20]. Ghasemi et al. [2] investigated numerically the influence of the magnetic field on buoyancy driven fluid flow in a square nanofluid enclosure filled with Al2O3-water. The results indicated that Rayleigh and Hartmann number had an opposite influence on heat transfer while the increasing of nanoparticle loading helps in augmentation of heat transfer. Another important study focus on using a different model of nanofluid properties presented by Lai and Yang [3] using Lattice-Boltzmann scheme. They found that different methods of nanofluid properties affect the Nusselt number value. Bhuwaneswari et al. [4] computed using a finite volume approach the natural convection with the magnetic field in a square enclosure. The major of this study is that the authors applied sinusoidal temperature distribution to both of the sidewalls and keeping the horizontal wall adiabatic. The results indicated that increasing Hartmann number...
reduces the heat transfer. Alam et al. [5] demonstrated the impact of aspect ratio and heat source power density on fluid flow circulation and heat transfer of free convection within rectangular enclosure heated and cooled partially from the vertical walls while the rest of them and others walls maintained adiabatic. They found that increases the aspect ratio augmented the heat transfer and it reaches its maximum value at aspect ratio equals to one. [6, 7] illustrated the double diffusive natural convection in tilted rectangular enclosure including the impact of a magnetic field and heat source. They concluded that heat source had a crucial impact on heat transfer. For more details about the previous works, the reader can be referring to [8-15]. Wang et al. [16] presented a comparison between the impact of Al₂O₃–H₂O and Ga₂O₃/H₂O and the radius under different Rayleigh numbers of heat transfer using a two-phase lattice Boltzmann method. The authors deduced that Ga₂O₃ enhances the heat transfer better than Al₂O₃-H₂O. Al-Farhany and Abdulkadhim [17] examined the conjugate problem in a square enclosure filled with porous media under various Rayleigh and Darcy numbers and they obtained that wall thickness effects on the transformation of the heat mode from convection into conduction which reduces the heat transfer. Barik and Al-Farhany [21] studied the influence of inclined baffle in nanofluid/porous square enclosure using COMSOL. Dutta et al. [18] studied the entropy generation in quadrant porous enclosure heated sinusoidally from its bottom wall. They concluded that entropy generation due to fluid friction is dominated at high values of Darcy number while they noticed the entropy formed due to heat transfer is the major influencer on low Darcy number. Torki and Etesami [19] experimentally studied the natural convection of a rectangular enclosure filled with SiO₂ at different nanofluid loading and enclosure inclination angles. They obtained that the nanofluid did not affect low concentration value and the inclination angle of a rectangular enclosure had a strong effect. In addition, the Nusselt number is increased as the inclination angle goes up. Graževičius et al. [20] studied experimentally and numerically using ANSYS 17.2 of natural convection for removing heat from the reactor using a passive system. Also, the natural convection in complex shapes had been reported by various researchers like [22-25].

Sheikholeslami and Chamkha [22] examined the free convection in a lid-driven enclosure filled with Fe₃O₄ with applied magnetic field and wavy wall. They obtained that increasing magnetic number, Rayleigh number, and nanofluid volume fraction increases the Nusselt number while Hartmann number had a reverse impact on Nusselt number. Sheikholeslami [23] studied the liquid metal due to natural convection in a wavy enclosure for various values of Rayleigh number, amplitude of wavy wall, and Hartmann number. It is worthy to mention here that the last parameters increasing lead to reduce the Nusselt number. Two important studies collect between MHD and wavy enclosure is presented in [24, 25] and they agreed with the previous mentioned results in the previous works of various researchers.

Besides that, various researchers dealt with the different shapes of an inner body located inside a regular simple enclosure shape like a square or rectangle that had been presented in Fig. 2. The inner body located within the enclosure had a wide range of applications like a solar collector, fuel cell, etc. The researchers among the world interested in understanding the effect of inner shapes like circular, triangular, elliptical, and wavy inside different shapes of the enclosure. Other researchers focus on the position of the inner body and change the direction vertically, horizontally and longitudinally. These are the main parameters that affect the heat transfer so that some of these studies are presented by [26-31]. Lee et al. [26] used an immersed boundary method to examine the changing of an inner cylinder located within square enclosure horizontally and longitudinally while another study by [27, 28] illustrates the vertical position on heat transfer. Ali et al. [29] studied the mixed convection due to rotating inner circular cylinder within square enclosure filled with air using ANSYS FLUENT. Roslan et al. [30] studied the heated circular cylinder located within the cold enclosure. The main important thing in this study is that the inner circular cylinder had sinusoidal temperature under unsteady conditions. We summarized the previous publications regarding the natural convection within simple shape, complex shapes and simple annulus enclosure in Table 1-3, respectively.

Finally, the present work concentrates on the inner body located inside the non-square enclosure.

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**Figure 1. Schematic diagram of simple shape of enclosure (square) [2]**

**Figure 2. Schematic diagram of the inner body within simple enclosure shape [27]**
Table 1 summarized the studies of natural convection inside a simple shape of enclosure

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objective</th>
<th>Enclosure shape</th>
<th>Software/model used</th>
<th>Conclusions</th>
<th>Special findings</th>
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</thead>
<tbody>
<tr>
<td>Ghasemi et al. [2]</td>
<td>MHD and nanofluid effect</td>
<td>Square</td>
<td>CVM using SIMPLE algorithm</td>
<td>Hartmann and Rayleigh numbers had an inverse effect on Nusselt number</td>
<td>Nanofluid effect may increases of decreases Nusselt number depending upon Rayleigh number</td>
</tr>
<tr>
<td>Lai and Yang [3]</td>
<td>Al2O3 nanofluid</td>
<td>Square</td>
<td>Lattice – Boltzmann method (LBM)</td>
<td>Nanofluid thermophysical models effect on the computation of the Nusselt number</td>
<td>LBM is recommended for the practical engineering applications</td>
</tr>
<tr>
<td>Bhuveswari et al. [4]</td>
<td>Sinusoidal temperature boundary conditions with MHD effect</td>
<td>Square</td>
<td>FVM</td>
<td>Increasing Hartmann number reduces the heat transfer</td>
<td>The phase deviation of applied Sinusoidal temperature effect on Nusselt number</td>
</tr>
<tr>
<td>Alam et al. [5]</td>
<td>Partial cooling/heating and aspect ratio</td>
<td>Rectangle</td>
<td>FEM</td>
<td>Nusselt number increases when the aspect ratio range from 0.5 – 10 beyond this value it reduces Nu as it goes up.</td>
<td>The maximum Nusselt number is achieved at aspect ratio equals to one.</td>
</tr>
<tr>
<td>Teamah et al. [6]</td>
<td>Double diffusive, Inclinations angle, MHD, buoyancy ratio</td>
<td>Inclined rectangle</td>
<td>CVM SIMPLER algorithm</td>
<td>Lowest and Highest value of Sherwood and Nu numbers were at 75 and 150</td>
<td>Sherwood number is not affected by heat absorption and generation.</td>
</tr>
<tr>
<td>El Qarnia et al. [9]</td>
<td>Phase change due to melting</td>
<td>Rectangle</td>
<td>FVM/FORTRAN</td>
<td>Two correlations had been developed.</td>
<td>The developed model can be used in phase change material</td>
</tr>
<tr>
<td>Nithyadevi et al. [15]</td>
<td>Effect of numbers of discrete heater, Prandtl number and heat generation</td>
<td>Rectangle</td>
<td>FVM</td>
<td>Increasing numbers of heater and Prandtl number enhance the heat transfer</td>
<td>Increasing heat generation reduces the heat transfer</td>
</tr>
<tr>
<td>Qi at a. [16]</td>
<td>Ra and nanofluid radius</td>
<td>Rectangle</td>
<td>Two-phase LBM</td>
<td>Small radius of nanofluid can enhance better than the big size</td>
<td>The augmentation of nanofluid is better at low Ra number</td>
</tr>
<tr>
<td>Al-Farhany and Abdulkadhim [17]</td>
<td>Conjugate problem in porous medium</td>
<td>Square</td>
<td>FEM/COMSOL</td>
<td>Increasing Ra and Da enhance the heat transfer</td>
<td>Increasing the conduction wall reduces the heat transfer</td>
</tr>
<tr>
<td>Dutta et al. [18]</td>
<td>Porous media, entropy generation with non-uniform bottom wall temperature</td>
<td>Quadrant</td>
<td>FEM</td>
<td>Increasing Ra and Da increase the heat transfer as mentioned in most of the publications</td>
<td>When the Darcy number is low, the heat transfer's entropy generation is higher while at the high Da, the entropy generation due to friction of fluids is higher</td>
</tr>
<tr>
<td>Torki et al. [19]</td>
<td>nanofluid, inclined enclosure</td>
<td>Rectangle</td>
<td>Experimental study</td>
<td>Increasing Rayleigh number increases Nu</td>
<td>Inclination effect is higher at low nanofluid loading</td>
</tr>
</tbody>
</table>

Table 2 summarized the studies of natural convection inside the complex shape of enclosure

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objective</th>
<th>Enclosure shape</th>
<th>Software/model used</th>
<th>Conclusions</th>
<th>Special findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sheikhholeslami et al. [22]</td>
<td>MHD, lid-driven cavity, ferro nanofluid</td>
<td>Wavy</td>
<td>FEM/FORTRAN</td>
<td>Increasing nanofluid loading, Rayleigh, and magnetic numbers increases the heat transfer while Hartmann increases to reduce it.</td>
<td>The authors studied the wavy top wall which is a little bit make a difference with the previous publications</td>
</tr>
<tr>
<td>Sheikhholeslami et al. [23]</td>
<td>MHD</td>
<td>Wavy</td>
<td>CVFEM</td>
<td>Increasing Hartmann number reduces Nu</td>
<td>Hartmann number is highly effects on fluid flow and heat transfer</td>
</tr>
<tr>
<td>Xiong, et al. [25]</td>
<td>Nanofluid/porous layers and MHD</td>
<td>Wavy</td>
<td>CVFEM</td>
<td>Ha increasing leads to reduction in the Nusselt number</td>
<td>Increasing Da leads to an improvement in the heat transfer</td>
</tr>
</tbody>
</table>
Table 3 summarized the studies of natural convection between inner body within regular (simple) enclosure shapes

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objective</th>
<th>Enclosure shape</th>
<th>Software/model used</th>
<th>Conclusions</th>
<th>Special findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lee et al. [26]</td>
<td>Position of inner circular cylinder in horizontal and diagonal direction</td>
<td>Circular cylinder within square enclosure</td>
<td>FVM</td>
<td>Increasing Ra number leads to enhance the Nusselt number.</td>
<td>It is noted that when the cylinder becomes closest to the corner or left walls, the eddies that located in the direction of the cylinder will be separated while one large eddies formed behind the cylinder.</td>
</tr>
<tr>
<td>Hussain and Hussein [27]</td>
<td>Vertical position of the inner cylinder, constant heat flux is considered for the cylinder</td>
<td>Circular cylinder within square enclosure</td>
<td>FVM</td>
<td>As Ra increases, Nu goes up</td>
<td>When the cylinder moves vertically upward, two inner cells (eddies) are formed below it.</td>
</tr>
<tr>
<td>Park et al. [28], [30] and [31]</td>
<td>Same the study of [27] except they applied isotherm hot temperature to the cylinder</td>
<td>Unsteady case study</td>
<td>Wavy temperature conditions inside a square cavity</td>
<td>COMSOL</td>
<td>Increasing the oscillation of the heat source leads to improve the heat transfer.</td>
</tr>
</tbody>
</table>

2. Triangular enclosure

This section describes different shapes of an inner body located inside a triangular enclosure. Schematic diagram of this case study is illustrated below in Fig. 3. Some of these studies presented by [32-38].

![Figure 3. Schematic diagram of the inner body in a triangular enclosure [32]](image)

Xu et al. [32] examined numerically the natural convection heat transfer between various shapes of the inner cylinder (circular, square, rhombic, and triangular) located inside the triangular enclosure tilted for various inclination angle. The inner cylinder is kept at uniform hot temperature while the triangular enclosure is cold. The governing equations had been solved numerically using a finite volume method and validated with the previously published work along the bottom wall. The results are crucial and indicate that increasing the Rayleigh numbers breaking down the symmetry of streamlines contours and concentrates the isotherms contours to the top between the gaps. Yu et al. [33] examined the influence of various values of Prandtl number on heat transfer between a circular cylinder located inside an inclined triangular enclosure. The equations of mass, energy, and momentum of fluid along with Boussinesq approximations had been solved using a finite volume scheme. The results indicate that a low Prandtl number less than 0.7 had no effect of fluid flow intensity and heat transfer while the inclination angle had strong effect on it. The authors proposed important empirical equations of Nusselt number in terms of Prandtl number. Selimefendigil and Öztop [34] examined the mixed convection between circular cylinder within right-angled triangular enclosure heated partially from its left vertical wall. The governing equations had been solved numerically using a finite element method. The results are important because of increasing Hartmann number reduces both of entropy generation and heat transfer while increasing nanofluid volume fraction and speed of rotating circular cylinder increases both of them. Yu et al. [35] studied the unsteady free convection between circular cylinder within triangular enclosure under various effects of Grashof number, aspect ratio (inner diameter), and inclination angle. They simulated this phenomenon using CFD code ANSYS Fluent 6.3 which is based upon a finite volume method. They developed a relation between Nusselt number as a function of Grashof number for different inner cylinder diameters and inclination angles. Wang et al. [36] examined the mixed convection within the different sizes of an inner rotating circular cylinder located within a triangular enclosure and the gap between them was filled with Ethylene glycol-silicon carbide nanofluid for different Rayleigh numbers. Fluent CFD code had been used to simulate the whole of this problem. The results of this paper agreed with the previously published works. Sourtiji et al. [37] examined the natural fluid flow between a circular cylinder within a triangular nanofluid enclosure using a control volume based on a finite element scheme. The nanofluid thermal thermo-physical properties like viscosity and thermal conductivity had been predicted using Brinkman and Maxwell-Garnett. It is obtained that adding void fraction of nanofluid had a remarkable impact at low Rayleigh numbers. Also, it had been observed that increasing inner circular cylinder diameter augments the heat transfer obviously. Another important investigation had been reported in Amrani et al. [38]. The authors studied the combined effect of radiation as well as free convective flow for triangular enclosure within rectangular body. As the most of researchers, finite volume method had been used to simulate this phenomenon under various Rayleigh numbers and aspect ratios.

3. Trapezoidal

This section summarized the convection heat transfer due to the density difference between the inner body located within the trapezoidal enclosure [39-43]. Schematic representation of this case is presented in Fig. 4.
Hussein et al. [39] investigated numerically the mixed convection heat transfer between the inner circular rotating cylinder immersed in the trapezoidal enclosure using COMSOL, which is based upon the finite element scheme. The gap between the inner body and the enclosure is divided into two layers: the upper layer is filled with Copper-water nanofluid, while the lower layer is composed of the same nanofluid immersed in a saturated porous medium. The authors studied the influences of many dimensionless parameters such as Rayleigh and Darcy numbers, nanofluid void fraction, as well as many geometrical parameters such as undulations number of a bottom wavy wall, inner body's diameter and rotational speed, and the thickness of the porous layer. The results were crucial and indicated that the increasing of Rayleigh, Darcy numbers, nanofluid void fraction, and inner body's diameter rotational leads to augmentation in the local Nusselt number, and that means enhancement of the heat transfer. However, the behavior of other parameters is inverse which means as the layer of the porous medium and the number of undulations goes up, the heat transfer is reduced. Esam et al. [40] examined the natural convection heat transfer in a trapezoidal enclosure filled with multilayers using the finite element method. The enclosure is partially heated from the bottom wall while the top wall is kept at isothermal cold temperature. The two inclined walls, as well as the inner circular cylinder and the rest length of the bottom wall, are assumed adiabatic. The upper layer is filled with Ag-water nanofluid, while the lower layer filled with porous media saturated with the same nanofluid. The results had been validated with the previously published works and the agreement was good. The results indicate that increasing nanofluid, Darcy, and Rayleigh numbers increases the fluid flow intensity and the heat transfer rate. However, the behavior of porous layer thickness is completely reversed. Khan et al. [41] explained the mixed convection heat transfer between the inner heated circular cylinder rotating counter-clockwise located in a trapezoidal enclosure. The trapezoidal enclosure is kept adiabatic at its top and bottom wall while the two inclined walls are kept at cold temperatures. They made a comparison between the influence of the rotating cylinder and a motionless cylinder in a square enclosure. Their results confirmed that the rotating cylinder as well as the inclination angle of the sidewall effect significantly on the heat transfer. Selimefendigil [42] demonstrated numerically the natural convection between different shapes of the inner conductive body within a trapezoidal enclosure filled with different shapes of nanoparticle (blade, spherical, and cylindrical). The top and bottom walls are adiabatic while the left and right vertical walls are kept at isothermal hot and cold temperature respectively. The authors used finite element scheme to solve the governing equations and the validation seems good. The parameters of this study were Rayleigh number, thermal conductivity ratio, solid volume fraction, shapes of the nanoparticle and the inclination angle. It is worthy to mention that they conclude the shape of nanoparticle change obviously the heat transfer and the cylindrical shape is recommended for better Nusselt number and better enhancement in heat transfer. Ahmed et al. [43] visualized numerically using finite element techniques the heat lines in a nanofluid trapezoidal enclosure separated by a porous divider. The top wall is kept cold while a non-uniform temperature profile is applied to the bottom wall. The side walls are adiabatic. The parameters of this study are Rayleigh and Darcy number as well as nanofluid volume fraction as physical parameters. Also, many geometrical parameters had been investigated such as porous layer thickness and its positions as the authors moved it vertically upwards and downwards. The major findings were increasing the thickness of porous divider with the Rayleigh numbers augments the heat transfer. Also, the position of the porous divider is significant at low Rayleigh number while at higher Rayleigh number, the Nusselt number will be at the minimum values when the divider moved vertically downward.

We summarized the studies of the inner body within triangular and trapezoidal enclosure in Table 4.

<table>
<thead>
<tr>
<th>Ref</th>
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<th>Software/ model used</th>
<th>Conclusions</th>
<th>Special findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xu et al. [32]</td>
<td>Laminar free convection, effect of Ra, aspect ratio and inclination angle</td>
<td>Cylinder inside triangular enclosure</td>
<td>CVM</td>
<td>At constant aspect ratio, the inclination angle and Ra effect significantly on Nusselt number.</td>
<td>Correlation of Nusselt number as a function of Ra for each value of aspect ratio</td>
</tr>
<tr>
<td>Yu et al. [31]</td>
<td>Effect of Prandtl number</td>
<td>Cylinder inside coaxial triangular</td>
<td>FVM</td>
<td>Inclination angle had a strong impact on Nu</td>
<td>Unique effect of low Prandtl number Nu while Pr≥0.7 it does not affect Nu</td>
</tr>
<tr>
<td>Selimefendigil and Oztog [34]</td>
<td>Mixed convection, MHD, nanofluid, entropy</td>
<td>Rotating insulated cylinder inside triangular enclosure</td>
<td>FEM</td>
<td>Increasing nanofluid concentrations and rotating lead to an increasing in total entropy and Nusselt number</td>
<td>Hartmann number increasing leads to a reduction in both the entropy and Nusselt number</td>
</tr>
<tr>
<td>Yu et al. [35]</td>
<td>Unsteady natural convection</td>
<td>Cylinder inside coaxial triangular</td>
<td>ANSYS Fluent</td>
<td>Correlations of Nusselt number had been developed.</td>
<td>Nusselt number history had been presented</td>
</tr>
<tr>
<td>Wang et al. Wang et al. [36]</td>
<td>Mixed convection, nanofluid</td>
<td>Cylinder inside coaxial triangular</td>
<td>ANSYS Fluent</td>
<td>Increasing Ra and nanofluid volume fraction improve the Nu</td>
<td>Rotational velocity effect significantly on Nu</td>
</tr>
</tbody>
</table>

![Figure 4. Schematic diagram of the inner body in a triangular enclosure [39]](image-url)
Sourtiji et al. [37]  Laminar buoyancy driven flow, nanofluid  Cylinder inside triangular enclosure  CVFEM  Nanoparticle improve the heat as mentioned in most of the studies  Maxwell–Garnett (MG) and Brinkman models had been used to simulate the nanofluid presence

Amrani et al. [38]  Radiation and natural convection effect  Rectangle inside triangular enclosure  FVM  Decreasing the aspect ratio and increasing the Ra number enhances the heat transfer  Thermal radiation promotes the heat transfer

Hussein et al. [39]  Mixed convection with multi-layer system  Rotating cylinder in trapezoidal enclosure with sinusoidal bottom wall  FEM  The increasing the size and the rotation speed of the inner cylinder in addition to increases the Da, R and nanofluid loading will improve the heat transfer  Increasing porous thickness and number of undulation of the bottom wall reduce the heat transfer

Esam et al. [40]  Free convection with multi-layer system  Fixed adiabatic cylinder within trapezoidal enclosure  FEM  The results indicate that increasing Ra, Da and nanofluid loading enhance the heat transfer  Increasing porous thickness reduces the heat transfer

Khan et al. [41]  Mixed convection, air  Rotating cylinder in trapezoidal enclosure  FEM  Grashof number for large inclination angle is very strong  Rotating speed of the inner cylinder and inclination angle of the trapezoidal wall effect highly on the Nusselt number

Selimefendigil [42]  Natural convection with different shapes of nanoparticles (blades, spherical and cylindrical)  Different shapes of inner body inside trapezoidal  FEM  Effect of Ra, thermal conductivity ratio, nanofluid loading and shapes of nanoparticle on Nu had been discussed  Cylindrical nanoparticle gives better performance

Ahmed et al. [43]  Visualization of heatlines of free convection  Trapezoidal enclosure divided by porous medium partition  FEM  Increasing Ra, nanofluid volume fraction and Darcy number augments the heat transfer  Porous position equals to 0.5 gives the better heat transfer

4. Parallelogrammic

This section summarized the convection heat transfer due to the density difference between the inner body located within the parallelogrammic enclosure [44-48]. The computational domain of these shapes is inserted in Fig. 5.

Hussein [44] investigated numerically the influence of the position of an inner circular cylinder located inside a parallelogrammic enclosure filled with air using a finite volume method. The inner circular cylinder is kept at a hot temperature while both vertical walls are cold. The top and bottom walls are adiabatic. The effect of Rayleigh number, the inclination angle of the vertical wall, and the inner circular cylinder position had been taken into account and examined their effect on fluid flow strength and the heat transfer. It was obtained that the maximum flow strength will be when the inclination angle is zero i.e., for square enclosure when the cylinder moves upwards by +0.1. It was also obtained that when the cylinder moves downward will have greater Nusselt number than moving upwards. Majdi et al. [45] examined the natural convection between the hot circular cylinder immersed in a nanofluid parallelogrammic enclosure. The finite element had been used to solve the governing equations of heat transfer and fluid flow numerically. The validation was in good agreement with the previous publishing works. The results indicated that the increase of nanofluid volume fraction and Rayleigh number enhances the heat transfer especially if the inner circular cylinder moves vertically downwards until it reaches -0.1.

Chamkha [46] examined numerically the conjugate (conductive-natural and forced) convection within the parallelogrammic enclosure separated by solid partition using the finite volume method. The influence of various parameters such as Richardson number, the inclination angle of the enclosure from the cavity left vertical wall as well as the thermal conductivity ratio. The results were crucial and all of the mentioned parameters affect the heat transfer rate. Baihi [47] used a finite volume scheme to simulate free convection under transient conditions within a parallelogrammic enclosure filled with air. The enclosure is partially heated from its left vertical wall while cold temperature conditions are applied to the right wall. The top and bottom walls are kept adiabatic. Hussain et al. [48] simulated the free convection in a parallelogrammic enclosure containing a volumetric source under various inclination angles. The enclosure is heated non-uniformly from its left wall while the right wall is at isotherm cold temperature. The top and bottom walls are adiabatic.

![Figure 5. Schematic diagram of the inner body in a parallelogrammic enclosure [45]](image)

5. Rhombic

This section summarized the convection heat transfer due to the density difference between the inner body located within rhombic enclosure [49-53]. The present case is illustrated in Fig. 6.

Anandalakshmi and Basak [49] examined numerically the entropy generation and natural convection in a rhombic enclosure filled with saturated porous medium for different heating situation and inclination angle. Different uniform temperatures applied on the bottom and top walls where the bottom wall is warmer. Adiabatic conditions are considered to
both inclined vertical walls. The finite element scheme had been used to solve the continuity, momentum, and energy equations. Choi et al. [50] demonstrated the position of an inner circular cylinder located within the rhombic enclosure subjected to transient conditions using the immersed boundary scheme. The simulation of this study was done under various dimensionless parameters, which are Rayleigh number and inner cylinder locations, which is changed vertically upwards and downwards. The results indicated that when the cylinder moves up, two circulations formed below it, and the Rayleigh number had a significant effect on the maps of streamlines and isotherms. Another important study was presented by Hosseinjani and Nikfar [51] focused on the natural convective fluid flow between two horizontal circular cylinders located within the nanofluid rhombic enclosure. The impact of symmetry, asymmetry, instability, and stability of Cu-O nanofluid under various Rayleigh numbers had been explained. The impact of other parameters had been included such as diameter and the distance of inner cylinder, nanoparticle void fraction. The results reported that increasing the distance leads to an increase in transient asymmetric flow. Dogonchi et al. [53] investigated numerically the natural convection between the circular cylinder within a partially heated rhombic enclosure using CVFEM. The influence of nanoparticle shape factor, nanoparticle volume fraction, and had been examined. It is obtained the platelet shape had a better heat transfer rate.

The studies of the inner body within Parallelogrammic and Rhombic enclosure have been summarized in Table 5.

![Figure 6. Schematic diagram of the inner body in a Rhombus enclosure [50]](image)

**Table 5 summarized the studies of natural convection between inner body within Parallelogrammic and Rhombic enclosure**

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<thead>
<tr>
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<th>Software/ model used</th>
<th>Conclusions</th>
<th>Special findings</th>
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</thead>
<tbody>
<tr>
<td>Hussein [44]</td>
<td>Natural convection in Parallelogrammic enclosure for different vertical locations</td>
<td>Cylinder inside Parallelogrammic enclosure filled with air</td>
<td>FVM</td>
<td>Increasing Ra leads to enhance the heat transfer and increases the fluid flow strength. In addition, it is recommended to incline the vertical wall 15 for better fluid strength.</td>
<td>For better fluid flow strength and heat transfer rate, it is recommended to move the inner cylinder towards the bottom wall vertically.</td>
</tr>
<tr>
<td>Majdi et al. [45]</td>
<td>Natural convection in Parallelogrammic enclosure for different vertical locations</td>
<td>Cylinder inside Parallelogrammic enclosure filled with nanofluid</td>
<td>FEM</td>
<td>Ra and nanofluid enhance the heat transfer and fluid flow strength</td>
<td>Increasing inclination angle enhances the heat transfer</td>
</tr>
<tr>
<td>Chamkha [46]</td>
<td>Mixed convection</td>
<td>Parallelogrammic with solid partition</td>
<td>FVM</td>
<td>The Thermal conductivity ratio, Ri number, inclination angle and the direction of movement of the left vertical wall upward and downward effect on heat transfer and fluid dynamics</td>
<td>Mean skin friction coefficient when the wall moves downward was higher than if it moves upward</td>
</tr>
<tr>
<td>Baiti [47]</td>
<td>Unsteady buoyancy in buildings</td>
<td>Parallelogrammic enclosure filled with air</td>
<td>FVM</td>
<td>The influence of Ra number and the slope of the building effect on the building cooling</td>
<td>The authors presented data for analysis of building including fluid flow and heat transfer</td>
</tr>
<tr>
<td>Hussain et al. [48]</td>
<td>Free convection, heat source with non-uniform left sidewall</td>
<td>Parallelogrammic enclosure filled with air</td>
<td>FVM</td>
<td>Internal and external Rayleigh number effect on heat transfer.</td>
<td>Increases inclination angle in positive direction leads to a reduction in fluid flow strength when it increases in the negative direction; the fluid flow circulation becomes larger.</td>
</tr>
<tr>
<td>Anandalakshmi and Basak [49]</td>
<td>Free convective flow with entropy generation with two different locations (cases) of the heat sources</td>
<td>Rhombic filled with porous medium</td>
<td>FEM</td>
<td>Rhombic with inclination angle 30 is recommended to usage as it gives minimum entropy generation. However, it gives less heat transfer enhancement</td>
<td>Case 1 is better in energy efficiency compared to case 2 as the latter produces much irreversibility more than case 1</td>
</tr>
</tbody>
</table>
6. Elliptical

This section summarized the convection heat transfer due to the density difference between the inner body located within the elliptical enclosure [54-58] as shown in Fig. 7. Sheikholeslami et al. [54] examined the natural convection between the inner elliptical body within a circular cylinder enclosure filled with nanofluid. The results explained that increasing Rayleigh number, nanoparticle, and inclination angle leads to an increase in the Nusselt number. Zhang et al. [55] investigated the natural convection between the hot elliptical inner body inside the cold square enclosure using a variational multiscale element scheme. The parameters were the major axis of the inner ellipse, Rayleigh number, and the inclination angle of the square enclosure. The results confirmed that inner body size, as well as the angle of inclination, had a noticeable impact on fluid flow. Sheikholeslami et al. [56] examined free convection, thermal radiation as well as a magnetic field between elliptical inner body within elliptical enclosure filled with nanofluid. Kefayati and Tang [57] examined numerically the inner cylinder or elliptical inner body inside a square enclosure by the lattice Boltzmann method. Abdulkadhim [58] demonstrated the free convection heat transfer within the elliptical enclosure with an inner circular cylinder. The gap was filled with nanofluid. The influence of the magnetic field, Rayleigh number, heat coefficient had been examined and addressed.

7. Wavy enclosure

This section summarized the convection heat transfer due to the density difference between the inner body located within wavy enclosure [24, 52, 58-62]. As an illustrative example for this case is indicated in Fig. 8.

One of the interesting investigation that presented by Dogonchi [52] which concluded the inner rhombic body within wavy enclosure filled with Fe$_3$O$_4$. Control volume based upon the finite element method had been used in the simulation. Magnetic field dependent upon new viscosity model is employed. Many parameters had been included in the study like Rayleigh and Hartmann number, radiation parameters, aspect ratio, and nanoparticle shape factor (platelet, cylindrical, and spherical). The results highlighted that the increase of aspect ratio when the Hartmann number remains constant will reduce the rate of heat transfer. Jabbar et al. [59] examined the wavy interface on heat transfer between square enclosure divided into two layers, nanofluid/porous layer as well as a non-newtonian layer for various undulation number of wavy and Rayleigh numbers. Hatami and Safari [60] used the finite element method to solve the free convection between the circular cylinder inside the wavy nanofluid enclosure. Boulahia et al. [61] modeled the inner hot and cold cylinders inside the wavy enclosure. Abdulkadhim et al. [62] illustrated the multi-layer system between the wavy inner cylinder within a wavy enclosure using a finite element scheme under various inner cylinder location and different undulations numbers. It can be seen in Table 6 which summarized the studies of elliptical and wavy enclosure with the internal body.
Table 6 summarized the studies of natural convection between inner body within elliptical and wavy enclosure

<table>
<thead>
<tr>
<th>Ref</th>
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<th>Software/model used</th>
<th>Conclusions</th>
<th>Special findings</th>
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<tbody>
<tr>
<td>Sheikholeslam i et al. [54]</td>
<td>Natural convection with nanofluid</td>
<td>Circular enclosure with inner elliptical body</td>
<td>CVFEM</td>
<td>Increasing of nanofluid loading, Rayleigh number and inclination angle increases the Nusselt number</td>
<td>Increasing Ra reduces the enhancement of heat transfer.</td>
</tr>
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<td>Zhang et al. [55]</td>
<td>Natural convection</td>
<td>Elliptical body inside tilted square enclosure</td>
<td>FEM</td>
<td>The size of inner ellipse, as well as Ra, is highly affected on Nu</td>
<td>The effect of inclination angle is small on Nu</td>
</tr>
<tr>
<td>Sheikholeslam i et al. [56]</td>
<td>Natural convective flow, MHD, nanofluid</td>
<td>Elliptical enclosure with internal elliptical body</td>
<td>CVFEM</td>
<td>Increasing of the inclination angle of the inner elliptical body increases the heat transfer rate</td>
<td>Derivation of formula of Nu</td>
</tr>
<tr>
<td>Kefayati and Tang [57]</td>
<td>Natural convection</td>
<td>Circle and elliptical inner body inside square enclosure</td>
<td>LBM</td>
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<td>As Bingham number goes up, heat transfer goes down</td>
</tr>
<tr>
<td>Abdulkadhim [58]</td>
<td>Natural convection, MHD, heat generation/absorption and nanofluid</td>
<td>Circular body in a nanofluid elliptical enclosure</td>
<td>FEM</td>
<td>Hartmann number and Ra had an inverse effect on Nu</td>
<td>The change of inner cylinder horizontally effect on the heat transfer characteristics</td>
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<td>Jabbar et al. [59]</td>
<td>Free convection with wavy wall</td>
<td>Square enclosure with wavy wall</td>
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<td>Hatami and Safari [60]</td>
<td>Natural convection and nanofluid</td>
<td>Internal cylinder inside wavy enclosure</td>
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<td>The location of inner cylinder effect on heat transfer</td>
<td>The central location gives better heat transfer characteristics</td>
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<tr>
<td>Boulahia et al. [61]</td>
<td>Natural convection</td>
<td>Cylinder inside wavy enclosure</td>
<td>FVM</td>
<td>Ra and nanofluid increasing leads to increases the heat transfer</td>
<td>Increasing number of undulations and reduction in the amplitude of the wavy surface leads to augmentation in heat transfer</td>
</tr>
<tr>
<td>Abdulkadhim et al. [62]</td>
<td>Natural convection with multilayer system</td>
<td>Wavy internal body within wavy enclosure</td>
<td>COMSOL</td>
<td>Number of corrugated effect are small</td>
<td>The inner body position effect on heat and fluid flow</td>
</tr>
</tbody>
</table>

8. Governing Equation

Finally, it is important to insert the governing equations used in this specific subject of natural convection heat transfer within enclosure filled with pure fluid, porous medium, and nanofluid [63].

\[
\frac{\partial v}{\partial x} = -\frac{k}{\mu} \frac{\partial^2 p}{\partial x \partial y} + Ra \frac{\partial T}{\partial x} \quad (7)
\]
\[
U \frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \quad (8)
\]

8.3. Nanofluid

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (9)
\]
\[
\rho_n (U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial x} + \mu_n (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) \quad (10)
\]
\[
\rho_n (U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial y} + \mu_n (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) + g \left[ (1 - \psi) (\rho \beta \gamma)_{nf} + \psi (\rho \beta \gamma_s) \right] (T - T_c) \quad (11)
\]
\[
U \frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} = \alpha_n (\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}) \quad (12)
\]

9. Conclusion

This paper presents a comprehensive literature review of the most published papers in the field of natural convection between inner bodies located inside different complex enclosure shapes. The main conclusions are:
The inner bodies inside trapezoidal, parallelogrammic enclosure are very limited, and more investigation should be done.

The studies regarding wavy enclosure are limited in a comparison with other simple shapes of enclosure despite its important applications in electronic equipment.

There are limitations in the studies of natural convection between the inner body located in a wavy enclosure.

Multi-layers system inside a wavy enclosure is limited as most of the recent studies focus on nanofluid, porous media filled the enclosure but there are serious limitations when the nanofluid/porous media filled the space.

Dufour and Soret effect on natural flow for the multi-layer system are not investigated yet in full-details.

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REFERENCES


