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Differential Sandwich Results For Univalent Functions

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Abstract :

In the present paper, we obtain some subordination and superordination Results involving the integral operator T_α for certain normalized analytic functions in the open unit disk. These results are applied to obtain sandwich results.

Keywords : Analytic function, differential subordination ,superordination ,sandwich sheorem, dominant, subordinant, integral operator.

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1. Introduction:

Let $H = H(U)$ be the class of analytic functions in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$. For n a positive integer and $a \in \mathbb{C}$, let $H[a, n]$ be the subclass of the function $f \in H$ consisting of functions of the form :

$$f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \dots \quad (a \in \mathbb{C}, n \in \mathbb{N}).$$

Also, let A be the subclass of H consisting of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \quad (1.1)$$

Let $f, g \in H$. The function f is said to be subordinate to g , or g is said to be superordinate to f , if there exists a Schwarz function w analytic in U with $w(0) = 0$ and $|w(z)| < 1$ ($z \in U$), such that $f(z) = g(w(z))$, in such a case, we write $f < g$ or $f(z) < g(z)$ ($z \in U$). If g is univalent in U , then $f < g$ if and only if $f(0) = g(0)$ and $f(U) \subset g(U)$.

Let $p, h \in H$ and $\psi(r, \delta, t; z): \mathbb{C}^3 \times U \rightarrow \mathbb{C}$. If p and $\psi(p(z), zp'(z), z^2 p''(z); z)$ are univalent functions in U and if p satisfies the second-order differential superordination

$$h(z) < \psi(p(z), zp'(z), z^2 p''(z); z), (z \in U), \quad (1.2)$$

then p is called a solution of the differential superordination (1.2). (If f is subordinate to g then g is superordinate to f). An analytic function q is called subordinant, of the differential superordination if $q < p$ for all the functions p satisfying (1.2).

An univalent subordinant $\tilde{q}$ that satisfies $q < \tilde{q}$ for all the subordinants q of (1.2) is called the best subordinant. Miller and Mocanu [9] have obtained sufficient conditions on the functions h, q and ψ for which the following implication holds :

$$h(z) < \psi(p(z), zp'(z), z^2 p''(z); z) \rightarrow q(z) < p(z). \quad (1.3)$$

For $f \in A$ Al-Shaqsi [2] defined the following integral operator $T_\alpha f(z)$ defined by $T_\alpha f(z) = z + \sum_{n=2}^{\infty} \frac{(\alpha)n-1}{(c)n-1} a_n z^n$. (1.4)

Moreover, from (1.4), it follows that

$$z(T_{\alpha+1} f(z))' = c\alpha T_{\alpha+1} f(z) - (c\alpha - 1) T_\alpha f(z). \quad (1.5)$$

Ali et al.[1] obtained sufficient conditions for certain normalized analytic functions to satisfy : $q_1(z) < \frac{zf'(z)}{f(z)} < q_2(z)$,

where q_1 and q_2 are given univalent functions in U with $q_1(0) = q_2(0) = 1$. Also, Tuneski [13] obtained sufficient conditions for

starlikeness of f in terms of the quantity $\frac{f'(z)f(z)}{(f'(z))^2}$. Recently,

Shanmugam et al. [11,12], Goyal et al. [8], Atshan and Abbas [3], Atshan and Jawad [5], Atshan and Kazim [6], and Atshan and Badawi [4] also obtained sandwich results for certain classes of analytic functions, for different conditions.

The main object here to find sufficient conditions for certain normalized analytic functions f to satisfy:

$$q_1(z) < \left(\frac{T_{\alpha+1} f(z)}{z}\right)^\delta < q_2(z)$$

and

$$q_1(z) < \left(\frac{pT_{\alpha+1} f(z) + (1-p)T_\alpha f(z)}{z}\right)^\delta < q_2(z),$$

where q_1 and q_2 are given univalent functions in U with $q_1(0) = q_2(0) = 1$.

2. Preliminaries: In order to prove our subordinations and superordinations results, we need the following definitions and lemmas.

Definition 2.1 [9] : Let Q the set of all functions $f(z)$ that are analytic and injective on $\bar{U} / E(f)$, where

$$\bar{U} = U \cup \{z \in \partial U\}, \text{ and } E(f) = \{z \in \partial U : \lim_{z \rightarrow z} f(z) = \infty\},$$

and are such that $f'(z) \neq 0$ for $z \in \partial U / E(f)$. Further, let the subclass of Q for which $f(0) = a$ be denoted by $Q(a)$, and $Q(0) = Q_0, Q(1) = Q_1 = \{f \in Q : f(0) = 1\}$.

Lemma 2.1 [9] : Let q be univalent in the unit disk U and let θ and ϕ be analytic in a domain D containing $q(U)$ with $\phi(w) \neq 0$ when $w \in q(U)$. Set $Q(z) = zq'(z)\phi(q(z))$ and $h(z) = \theta(q(z)) + Q(z)$. suppose that,

(i) $Q(z)$ is starlike univalent in U ,

(ii) $\operatorname{Re} \left\{ \frac{zh'(z)}{Q(z)} \right\} > 0$ for $z \in U$.

If p is analytic in U with $p(0) = q(0)$, $p(U) \subseteq D$ and

$$\theta(p(z)) + zp'(z)\phi(p(z)) < \theta(q(z)) + zp'(z)\phi(q(z)), \quad (2.1)$$

then $p < q$ and q is the best dominant of (2.1).

Lemma 2.2 [10] : Let q be convex univalent function in U and let $\alpha \in \mathbb{C}$, $\beta \in \mathbb{C} / \{0\}$ and suppose that

$$\operatorname{Re} \left\{ 1 + \frac{zq''(z)}{q'(z)} \right\} > \max \{0, -\operatorname{Re} \left(\frac{\alpha}{\beta} \right)\}.$$

If p is analytic in U , and

$$\alpha p(z) + \beta zp'(z) < \alpha q(z) + \beta zq'(z), \quad (2.2)$$

then $p < q$ and q is the best dominant of (2.2).

Lemma 2.3 [10] : Let q be convex univalent in U and $q(0) = 1$. Let $\beta \in \mathbb{C}$. Further assume that $\operatorname{Re}(\beta) > 0$. If $p \in H[q(0), 1] \cap Q$ and $p(z) + \beta zp'(z)$ is univalent in U , then

$$q(z) + \beta zq'(z) < p(z) + \beta zp'(z), \quad (2.3)$$

which implies that $q < p$ and q is the best subordinant of (2.3).

Lemma 2.4 [7] : Let q be convex univalent in the unit disk U and let θ and ϕ be analytic in a domain D containing $q(U)$. Suppose that

(i) $\operatorname{Re} \left\{ \frac{\theta'(q(z))}{\phi(q(z))} \right\} > 0$ for $z \in U$,

(ii) $Q(z) = zq'(z)\phi(q(z))$ is starlike univalent in $z \in U$.

If $p \in H[q(0), 1] \cap Q$ with $p(U) \subseteq D$, $\theta(p(z)) + zp'(z)\phi(p(z))$,

is univalent in U and

$$\theta(q(z)) + zq'(z)\phi(q(z)) < \theta(p(z)) + zp'(z)\phi(p(z)), \quad (2.4)$$

then $q < p$ and q is the best subordinant of (2.4).

3. Subordination Results

Theorem 3.1 : Let q be convex univalent function in U with $q(0) = 1$,

$0 \neq \eta \in \mathbb{C}$, $\delta > 0$ and suppose that q satisfies :

$$\operatorname{Re} \left\{ 1 + \frac{zq''(z)}{q'(z)} \right\} > \max \{0, -\operatorname{Re} \left(\frac{\delta}{\eta} \right)\}. \quad (3.1)$$

If $f \in A$ satisfies the subordination

$$1 - \eta \left(c\alpha - 1 \right) \left(\frac{T_{\alpha+1} f(z)}{z} \right)^{\delta + \eta} \left(c\alpha - 1 \right) \left(\frac{T_{\alpha+1} f(z)}{z} \right)^\delta \left(\frac{T_\alpha f(z)}{T_{\alpha+1} f(z)} \right) < q(z) + \frac{\eta}{\delta} z q'(z), \quad (3.2)$$

$$\text{then } \left(\frac{T_{\alpha+1} f(z)}{z} \right)^\delta < q(z) \quad (3.3)$$

and q is the best dominant of (3.2).

Proof : Consider a function $p(z)$ by

$$p(z) = \left(\frac{T_{\alpha+1} f(z)}{z} \right)^\delta. \quad (3.4)$$

Differentiating (3.4) with respect to z logarithmically, we get

$$\frac{z p'(z)}{p(z)} = \delta \left(\frac{z T_{\alpha+1} f(z)'}{T_{\alpha+1} f(z)} - 1 \right). \quad (3.5)$$

Now, in view of (1.5), we obtain

$$\frac{z p'(z)}{p(z)} = \delta \left(c\alpha \left(\frac{T_{\alpha+1} f(z)}{T_{\alpha+1} f(z)} - 1 \right) + \left(\frac{T_{\alpha+1} f(z)}{T_{\alpha+1} f(z)} - 1 \right) \right).$$

Therefore

$$\frac{z p'(z)}{\delta} = \left(\frac{T_{\alpha+1} f(z)}{z} \right)^\delta \left(c\alpha \left(\frac{T_{\alpha+1} f(z)}{T_{\alpha+1} f(z)} - 1 \right) + \left(\frac{T_{\alpha+1} f(z)}{T_{\alpha+1} f(z)} - 1 \right) \right).$$

The subordination (3.2) from the hypothesis becomes

$$p(z) + \frac{\eta}{\delta} z p'(z) < q(z) + \frac{\eta}{\delta} z q'(z).$$

An application of Lemma (2.2) with $\beta = \frac{\eta}{\delta}$ and $\alpha = 1$, we obtain (3.3).

Putting $q(z) = \left(\frac{1+z}{1-z} \right)$ in Theorem (3.1), we obtain the following corollary.

Corollary 3.1, Let $0 \neq \eta \in \mathbb{C}$, $\delta > 0$ and

$$\operatorname{Re} \left\{ 1 + \frac{2z}{1-z} \right\} > \max \left\{ 0, -\operatorname{Re} \left(\frac{\delta}{\eta} \right) \right\}.$$

If $f \in A$ satisfies the subordination

$$1 - \eta \left(c\alpha - 1 \right) \left(\frac{T_{\alpha+1} f(z)}{z} \right)^{\delta+\eta} + \eta \left(c\alpha - 1 \right) \left(\frac{T_{\alpha+1} f(z)}{z} \right)^\delta \left(\frac{T_\alpha f(z)}{T_{\alpha+1} f(z)} \right) < \left(\frac{1-z^2+2\frac{\eta}{\delta}z}{(1-z)^2} \right), \text{ then}$$

$$\left(\frac{T_{\alpha+1} f(z)}{z} \right)^\delta < \left(\frac{1+z}{1-z} \right),$$

and $q(z) = \left(\frac{1+z}{1-z} \right)$ is the best dominant.

Theorem 3.2 : Let q be convex univalent in U with $q(0) = 1$, $q(z) \neq 0$ ($z \in U$) and assume that q satisfies:

$$\operatorname{Re} \left\{ 1 - \frac{\delta}{\eta} + \frac{z q''(z)}{q'(z)} \right\} > 0, \quad (3.6)$$

where $\eta \in \mathbb{C} \setminus \{0\}$, $\lambda > 0$ and $z \in U$. Suppose that $-\eta z q'(z)$ is starlike univalent in U . If $f \in A$ satisfies:

$$\phi(\lambda, \delta, c, \eta; z) < \delta q(z) - \eta z q'(z), \quad (3.7)$$

where $\phi(\lambda, \delta, c, \eta; z) = \delta \left(\frac{p T_{\alpha+1} f(z) + (1-p) T_\alpha f(z)}{z} \right)^\delta -$

$$\eta \delta \left(\frac{p T_{\alpha+1} f(z) + (1-p) T_\alpha f(z)}{z} \right)^\delta \left(\frac{p T_\alpha f(z) + (1-p) T_{\alpha+1} f(z)}{p T_{\alpha+1} f(z) + (1-p) T_{\alpha+1} f(z)} - 1 \right), \quad (3.8)$$

$$\text{then } \left(\frac{p T_\alpha f(z) + (1-p) T_{\alpha+1} f(z)}{z} \right)^\delta < q(z), \quad (3.9)$$

and $q(z)$ is the best dominant of (3.7).

Proof : Consider a function $p(z)$ by

$$p(z) = \left(\frac{p T_{\alpha+1} f(z) + (1-p) T_\alpha f(z)}{z} \right)^\delta \quad (3.10)$$

by setting,

$$\theta(w) = \delta w \text{ and } \phi(w) = -\eta, w \neq 0$$

we see that $\theta(w)$ is analytic in $\mathbb{C}$, $\phi(w)$ is analytic in $\mathbb{C} \setminus \{0\}$ and that $\phi(w) \neq 0$, $w \in \mathbb{C} \setminus \{0\}$. Also, we get

$$Q(z) = z q'(z) \phi(q(z)) = -\eta z q'(z),$$

and

$$h(z) = \theta(q(z)) + Q(z) = \delta q(z) - \eta z q'(z).$$

It is clear that $Q(z)$ is starlike univalent in U .

$$\operatorname{Re} \left\{ \frac{z h'(z)}{Q(z)} \right\} = \operatorname{Re} \left\{ 1 - \frac{\delta}{\eta} + \frac{z q''(z)}{q'(z)} \right\} > 0.$$

By a straightforward computation, we obtain

$$\delta p(z) - \eta z p'(z) = \phi(\lambda, \delta, c, \Psi; z), \quad (3.11)$$

where $\phi(\lambda, \delta, c, \Psi; z)$ is given by (3.8) from (3.7) and (3.11), we have $\delta p(z) - \eta z p'(z) < \delta q(z) - \eta z q'(z)$. (3.12)

Therefore, By Lemma (2.1), we get $p(z) < q(z)$. By using (3.10), we obtain the result.

Putting $q(z) = \frac{1+Az}{1+Bz}$ ($-1 \leq B < A < 1$) in Theorem (3.2), we obtain The following Corollary.

Corollary 3.2 : Let $-1 \leq B < A \leq 1$ and

$$\operatorname{Re} \left\{ 1 - \frac{\delta}{\eta} + \frac{z 2B}{1+Bz} \right\} > 0,$$

where $\eta \in \mathbb{C} \setminus \{0\}$ and $z \in U$, if $f \in A$ satisfies

$\phi(\lambda, \delta, c, \eta; z) < \left(\delta \left(\frac{1+Az}{1+Bz} \right) - \eta z \frac{A-B}{(1+Bz)^2} \right)$ and $\phi(\lambda, \delta, c, \Psi; z)$ is given by (3.8).

$$\left(\frac{p T_{\alpha+1} f(z) + (1-p) T_\alpha f(z)}{z} \right) < \frac{1+Az}{1+Bz}$$

and $q(z) = \frac{1+Az}{1+Bz}$ is the best dominant.

4. Superordination Results

Theorem 4.1: Let q be convex univalent in U with $q(0) = 1$, $\delta > 0$ and $\operatorname{Re} \{\eta\} > 0$. Let $f \in A$ satisfies $\left(\frac{T_{\alpha+1} f(z)}{z} \right)^\delta \in H[q(0), 1] \cap Q$ and

$$(1 - \eta(c\alpha)) \left(\frac{T_{\alpha+1} f(z)}{z} \right)^\delta + \eta(c\alpha - 1) \left(\frac{T_{\alpha+1} f(z)}{z} \right)^\delta \left(\frac{T_\alpha f(z)}{T_{\alpha+1} f(z)} \right)$$

be univalent in U . If $q(z) + \frac{\eta}{\delta} z q'(z) <$

$$(1 - \eta(c\alpha)) \left(\frac{T_{\alpha+1} f(z)}{z} \right)^{\delta+} + \eta(c\alpha - 1) \left(\frac{T_{\alpha+1} f(z)}{z} \right)^\delta \left(\frac{T_\alpha f(z)}{T_{\alpha+1} f(z)} \right), \quad (4.1)$$

$$\text{then } q(z) < \left(\frac{T_{\alpha+1} f(z)}{z} \right)^\delta \quad (4.2)$$

and q is the best subordinator of (4.1).

Proof : Consider a function $p(z)$ by $P(z) = \left(\frac{T_{\alpha+1}f(z)}{z}\right)^\delta$. (4.3)

Differentiating (4.3) with respect to z logarithmically, we get

$$\frac{zp'(z)}{p(z)} = \delta \left(\frac{z(T_{\alpha+1}f(z))'}{T_{\alpha+1}f(z)} - 1 \right). \quad (4.4)$$

After some computations and using (1.5), from (4.4), we obtain

$$(1 - \eta(c\alpha)) \left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta + \eta(c\alpha - 1) \left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta \left(\frac{T_\alpha f(z)}{T_{\alpha+1}f(z)} \right) = p(z) + \frac{\eta}{\delta} zp'(z)$$

and now, by using Lemma (2.3), we get the desired result.

Putting $q(z) = \frac{1+z}{1-z}$ in Theorem (4.1), we obtain the following Corollary.

Corollary 4.1: Let $\delta > 0$ and $\text{Re}\{\eta\} > 0$. If $f \in A$ satisfies:

$$\left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta \in H[q(0), 1] \cap Q$$

and

$$(1 - \eta(c\alpha)) \left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta + \eta(c\alpha - 1) \left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta \left(\frac{T_\alpha f(z)}{T_{\alpha+1}f(z)} \right),$$

be univalent in U . If

$$\left(\frac{1 - z^2 + 2\frac{\eta}{\delta}z}{(1-z)^2} \right) <$$

$$(1 - \eta(c\alpha)) \left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta + \eta(c\alpha - 1) \left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta \left(\frac{T_\alpha f(z)}{T_{\alpha+1}f(z)} \right),$$

then

$$\left(\frac{1+z}{1-z} \right) < \left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta$$

and $q(z) = \frac{1+z}{1-z}$ is the best subordinator.

Theorem 4.2: Let q be convex univalent in U with $q(0) = 1$, and assume that q satisfies

$$\text{Re} \left\{ \frac{-\delta q'(z)}{\eta} \right\} > 0, \quad (4.5)$$

where $\eta \in \mathbb{C} \setminus \{0\}$ and $z \in U$.

Suppose that $-\eta zq'(z)$ is starlike univalent in U , let $f \in A$ satisfies :

$$\left(\frac{pT_{\alpha+1}f(z) + (1-p)T_\alpha f(z)}{z} \right) \in H[q(0), 1] \cap Q$$

and $\phi(\lambda, \delta, c, \eta; z)$ is univalent in U , where $\phi(\lambda, \delta, c, \eta; z)$ is given by (3.8). If $\delta q(z) - \eta zq'(z) < \phi(\lambda, \delta, c, \eta; z)$, (4.6)

$$\text{then } q(z) < \left(\frac{pT_{\alpha+1}f(z) + (1-p)T_\alpha f(z)}{z} \right)^\delta \quad (4.7)$$

and q is the best subordinator of (4.6).

Proof : Consider a function $p(z)$ by

$$p(z) = \left(\frac{pT_{\alpha+1}f(z) + (1-p)T_\alpha f(z)}{z} \right)^\delta. \quad (4.8)$$

By setting

$$\theta(w) = \delta w \text{ and } \phi(w) = -\eta, w \neq 0,$$

we see that $\theta(w)$ is analytic in $\mathbb{C}$, $\phi(w)$ is analytic in $\mathbb{C} \setminus \{0\}$ and

that $\phi(w) \neq 0$, $w \in \mathbb{C} \setminus \{0\}$. Also, we get

$$Q(z) = zq'(z) \phi(q(z)) = -\eta zq'(z).$$

It is clear that $Q(z)$ is starlike univalent in U ,

$$\text{Re} \left\{ \frac{\theta'(q(z))}{\phi(q(z))} \right\} = \text{Re} \left\{ \frac{-\delta q'(z)}{\eta} \right\} > 0.$$

By a straightforward computation, we obtain

$$\phi(\lambda, \delta, c, \eta; z) = \delta p(z) - \eta zp'(z), \quad (4.9)$$

where $\phi(\lambda, \delta, c, \eta; z)$ is given by (3.8). From (4.6) and (4.9), we have

$$\delta q(z) - \eta zq'(z) < \delta p(z) - \eta zp'(z).$$

Therefore, by Lemma (2.4), we get $q(z) < p(z)$. by using (4.8), we obtain the result.

Concluding the results of differential subordination and superordination we arrive at the following " sandwich results " .

5. Sandwich Results

Theorem 5.1 : Let q_1 be convex univalent in U with $q_1(0) = 1$, $\text{Re}\{\eta\} > 0$ and let q_2 be univalent in U , $q_2(0) = 1$ and satisfies (3.1), let $f \in A$ satisfies :

$$\left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta \in H[1, 1] \cap Q$$

and

$$(1 - \eta(c\alpha)) \left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta + \eta(c\alpha - 1) \left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta \left(\frac{T_\alpha f(z)}{T_{\alpha+1}f(z)} \right)$$

be univalent in U . If $q_1(z) + \frac{\eta}{\delta} zq_1'(z) <$

$$(1 - \eta(c\alpha)) \left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta + \eta(c\alpha - 1) \left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta \left(\frac{T_\alpha f(z)}{T_{\alpha+1}f(z)} \right) < q_2(z) + \frac{\eta}{\delta} zq_2'(z),$$

$$\text{then } q_1(z) < \left(\frac{T_{\alpha+1}f(z)}{z} \right)^\delta < q_2(z),$$

and q_1 and q_2 are respectively, the best subordinator and the best dominant.

Theorem 5.2: Let q_1 be convex univalent in U with $q_1(0) = 1$, and satisfies (4.5). Let q_2 be univalent in U $q_2(0) = 1$, satisfies (3.6), let $f \in A$ satisfies

$$\left(\frac{pT_{\alpha+1}f(z) + (1-p)T_\alpha f(z)}{z} \right)^\delta \in H[1, 1] \cap Q$$

and $\phi(\lambda, \delta, c, \eta; z)$ is univalent in U , where $\phi(\lambda, \delta, c, \eta; z)$ is given by (3.8). If

$$\delta q_1(z) - \eta z q_1'(z) < \phi(\lambda, \delta, c, \eta; z) < \delta q_2(z) - \eta z q_2'(z),$$

then

$$q_1(z) < \left(\frac{p T_{\alpha+1} f(z) + (1-p) T_{\alpha} f(z)}{z} \right)^{\delta} < q_2(z),$$

and q_1 and q_2 are respectively the best subdominant and the best dominant.

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