On Semi Feebly Open Set and its Properties

Authors Names	ABSTRACT
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1. Introduction

The topological idea from study this set is generalization the properties and using it's to prove the theorems. In [7] N. Leven (1963) gives the definition of semi open (*s*-open) set, semi closed (*s*-closed) set and studies the properties of it. He defined a set A named (*s*-open) set in topological space X if find an open set O : $O \subseteq A \subseteq \bar{O}$ where $\bar{O}$ denoted by the closure of O in X , the complement semi-open (*s*-open) set called semi-closed (*s*-closed) set. In (1971) S. G. Crossiey and S. K. Hildebrand defined the concept semi closure and they defined it, the semi closure of a set A in topological space X is the smallest semi-closed (*s*-closed) set contained A [2] and shortened by $Scl(A)$ or $\bar{A}^s$. The truth $\bar{A}^s$ is the intersection of all semi closed sets contained A , $\bar{A}^s \subseteq \bar{A}$ and $\overline{\bar{A}^s}^s = \bar{A}^s$. Maheswari and Tapi (1978) in [3] defined feebly closed (*f*-closed), feebly open (*f*-open) set. A set A in a topological space X named feebly open (*f*-open) set in X if find an open set V such that $V \subseteq A \subseteq \bar{V}^s$. A set A in a topological space X is feebly closed

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if it is complement is feebly open. Every open is (f -open) set, but the converse may be not true. Every closed is (f -closed) set, but the converse may be not true.

We will use the (tp - s) symbol to denote the topological space, (s -open) to semi open set, (s -closed) to semi closed set, (f -open) to feebly open set and (f -closed) to feebly closed set . wherever it is found in this paper.

2. Preliminaries

Definition(2.1)[7]: Assume that (X, τ) is a tp - s & $A \subseteq X$. Then A is named s -open in X if there exists $O \in \tau : O \subseteq A \subseteq \overline{O}$. Or equivalent [5], A called s -open in $X \Leftrightarrow A \subseteq \overline{A}^\circ$, equivalent $\overline{A} = \overline{A}^\circ$, the complement of s -open is named s -closed.

Definition(2.2)[7]: Let (X, τ) be tp - s & $A \subseteq X$ then A called s -closed in X if there exists a closed set F such that $F^\circ \subseteq A \subseteq F$, or equivalent[5], A is s -closed in $X \Leftrightarrow \overline{A}^\circ \subseteq A$, equivalent $A^\circ = \overline{A}^\circ$.

Definition(2.3)[5]: Let (X, τ) be tp - s & $A \subseteq X$, then the intersection of all s -closed subset of X contained A named (s -closure) of A and the union of all s -open subset of X contained A named (s -interior) of A and are shortened by $\overline{A}^s, A^{os}$ respectively.

Proposition (2.4)[7]: Let $\{A_\lambda\}_{\lambda \in \Lambda}$ be a family of s -open in a tp - s X then $\bigcup_{\lambda \in \Lambda} A_\lambda$ is s -open.

Proposition (2.5)[7]: Let X be a tp - s then the intersection of two s -open in X does not need to be s -open.

Example (2.6): Let $X = \{k, v, h\}, \tau = \{\{k\}, \{v\}, \{k, v\}, X, \emptyset\}$ then each of $\{k, h\}, \{v, h\}$ are s -open, but $\{k, h\} \cap \{v, h\} = \{h\}$ not s -open.

Definition(2.7)[4]: The intersection of every semi open subset of tp - s X contained a set A is named Semi kernel of A and shortened by ($S \ker(A)$).

Means that : $S \ker(A) = \bigcap \{U : U \text{ s-open and } A \subseteq U\}$.

Definition(2.8)[8]: A set A in a tp - s X called f -open in X if there exists an open set V such that $V \subseteq A \subseteq \overline{V}^s$, or equivalent, A set A called f -open in X if and only if $A \subseteq \overline{A}^\circ$, the complement of f -open is called f -closed that $\overline{A}^\circ \subseteq A$.

Remark(2.9)[6]: Let (X, τ) be tp - s & $A \subseteq \tau$ then A is f -open and A^c is f -closed.

But the converse is not true in general as in the next example.

Example (2.10): Assume that $X = \{1, 2, 3, 4, 5\}$ and $\tau = \{\emptyset, X, \{1\}, \{2, 4\}, \{1, 2, 4\}\}$ then, f -open sets are $\{\emptyset, X, \{1\}, \{2, 4\}, \{1, 2, 4\}, \{1, 2, 3, 4\}\}$, f -closed sets are $\{\emptyset, X, \{2, 3\}, \{1, 3, 5\}, \{3, 5\}, \{5\}\}$.

Take $A = \{1, 2, 3, 4\}$ is f -open, but it not open set & $A^c = \{5\}$ f -closed, but it is not closed.

Proposition (2.11)[9]: Assume that (X, τ) is a tp - s & A, B subsets of X then :

1. $\overline{A}^f$ f -closed .
2. $A \subseteq \overline{A}^f$.
3. A is (f -closed) $\Leftrightarrow A = \overline{A}^f$.
4. $A \subseteq B \Rightarrow \overline{A}^f \subseteq \overline{B}^f$.
5. If $\{A_\lambda\}_{\lambda \in \Lambda}$ be a collection of subset of X then $\overline{\bigcup_{\lambda \in \Lambda} A_\lambda}^f = \bigcup_{\lambda \in \Lambda} \overline{A_\lambda}^f$.
6. If $\{A_\lambda\}_{\lambda \in \Lambda}$ be a collection of subset of X then $\overline{\bigcap_{\lambda \in \Lambda} A_\lambda}^f \subseteq \bigcap_{\lambda \in \Lambda} \overline{A_\lambda}^f$.
7. $\overline{\overline{A}^f}^f = \overline{A}^f$.
8. $\overline{A}^f \subseteq \overline{A}$
9. $\overline{\overline{A}^f} = \overline{A}^f = \overline{A}$.
10. $\overline{A}^f = A \cup A'^f$.
11. $\overline{A}^f = A \cup \overline{A}^\circ$.
12. $x \in \overline{A}^f \Leftrightarrow$ any f -open G contained x , $A \cap G \neq \emptyset$.

Proposition(2.12)[9]: Let X be a tp -s & A, B subset of X where B f -open, If $x \in B$ and $A \cap B = \emptyset$ then $x \notin \overline{A}^f$.

Definition(2.13)[10]: Let X be a tp -s a subset A of X is said to be

- i. Dense or (every dense) in $X \Leftrightarrow \overline{A} = X$.
- ii. Nowhere dense or (non-dense) in X iff $(\overline{A})^\circ = \emptyset$.

Definition(2.14)[5]: Let (X, τ) be a tp -s and $A \subseteq X$, A is named preopen (p -open) if $A \subseteq \overline{A}^\circ$ and A^c is named pre closed (p -closed) that $\overline{A}^\circ \subseteq A$.

Lemma(2.15)[4]: Every singleton $\{x\}$ in a tp -s X is either nowhere dense or preopen.

3. The Main Results

Definition(3.1): Assume that (X, τ) is a tp -s then a subset A in a space X is named semi feebly open (sf -open) set in a space X if $A \subseteq U$ where U semi open set in X then $\overline{A}^f \subseteq U$. The complement of semi feebly open is called semi feebly closed (sf -closed) it is as follows $U \subseteq A^{\circ f}$ where U semi closed set in X .

Example (3.2): Let $X = \{k, v, h\}$, $\tau = \{X, \emptyset, \{k\}\}$ then

open set : $\{X, \emptyset, \{k\}\}$, closed set : $\{\emptyset, X, \{v, h\}\}$

s -open : $\{\emptyset, X, \{k\}, \{k, v\}, \{k, h\}\}$, s -closed : $\{\emptyset, X, \{v, h\}, \{h\}, \{v\}\}$

f -open : $\{\emptyset, X, \{k\}, \{k, v\}, \{k, h\}\}$, f -closed : $\{\emptyset, X, \{v, h\}, \{h\}, \{v\}\}$

sf -open : $\{\emptyset, X, \{v\}, \{h\}, \{v, h\}\}$

we notes that $\{k\}, \{k, v\}, \{k, h\}$ not sf -open because $\{k\} \subseteq \{k\}$ where $\{k\}$ s -open, but $\overline{\{k\}}^f = X \not\subseteq \{k\}$, $\{k\}$ is not sf -open.

$\{k, v\} \subseteq \{k, v\}$ where $\{k, v\}$ s -open, but $\overline{\{k, v\}}^f = X \not\subseteq \{k, v\}$, $\{k, v\}$ is not sf -open.

$\{k, h\} \subseteq \{k, h\}$ where $\{k, h\}$ s -open, but $\overline{\{k, h\}}^f = X \not\subseteq \{k, h\}$, $\{k, h\}$ is not sf -open.

Remark(3.3): Each f -closed is sf -open.

Proof : let A be f -closed set in a tp - s X . $A \subseteq U$, U s -open, A is (f -closed) set then $A = \overline{A}^f$ and $A = \overline{A}^f \subseteq U \Rightarrow A$ is (sf -open) set.

The converse of (Remark(3-3)) is not true in general, as in the next example shows:

Examl(3.4): Let $X = \{1, 2, 3, 4, 5\}$, $\tau = \{X, \emptyset, \{1\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4, 5\}\}$ and $A = \{1, 2, 4, 5\}$ then A is sf -open not f -closed.

Proof: The open sets are $\{X, \emptyset, \{1\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4, 5\}\}$,

the closed sets are $\{\emptyset, X, \{2, 3, 4, 5\}, \{1, 2, 5\}, \{2, 5\}, \{1\}\}$ and

s -open sets are $\{X, \emptyset, \{1\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4, 5\}, \{2, 3, 4\}, \{3, 4, 5\}, \{1, 2, 3, 4\}, \{1, 3, 4, 5\}\}$

Let $A = \{1, 2, 4, 5\}$ and $U = X$ which is s -open, then $A \subseteq U$

$$\overline{A}^f = \{1, 2, 4, 5\} \cup \overline{\overline{\{1, 2, 4, 5\}}^o} \quad [\text{Proposition (2.11)(11)}]$$

$$\overline{A}^f = \{1, 2, 4, 5\} \cup X = X \subseteq U = X \Rightarrow A \text{ } sf \text{-open set, but } A \text{ not } f\text{-closed because } \overline{\overline{\{1, 2, 4, 5\}}^o} = X \not\subseteq \{1, 2, 4, 5\}.$$

Remark(3.5): Every closed set is sf -open set.

Proof: Let A closed set $\Rightarrow A$ is f -closed by [Remark (1.15)] A is sf -open set.

But the converse of (Remark(3-5)) in general is not true as in our [Example (3.4)] shows :

$A = \{1, 2, 4, 5\}$ sf -open, $\overline{A} = X \neq A \Rightarrow \overline{A} \neq A$ then A is not closed.

The next diagram explains the relationship these types of sets.

Notes(3.6)

For each $tp\text{-}s$ $\emptyset, X$ are $sf\text{-open}$.

Every subset of discrete or indiscrete $tp\text{-}s$ is $sf\text{-open}$.

Every closed interval in (R, U) where U is usual topology is $sf\text{-open}$.

Proposition(3.7): Let X be $tp\text{-}s$ then the union of all $sf\text{-open}$ sets in X is also $sf\text{-open}$ set.

Proof: Let $\bigcup_{\lambda \in \Lambda} A_{\lambda} \subseteq U$, U semi-open in a topological space X then

$$A_{\lambda} \subseteq \bigcup_{\lambda \in \Lambda} A_{\lambda} \subseteq U \Rightarrow A_{\lambda} \subseteq U \Rightarrow \overline{A_{\lambda}}^f \subseteq U \Rightarrow \bigcup_{\lambda \in \Lambda} \overline{A_{\lambda}}^f \subseteq U$$

since $\{A_{\lambda}\}_{\lambda \in \Lambda}$ be a collection of all subset of X then $\overline{\bigcup_{\lambda \in \Lambda} A_{\lambda}}^f = \bigcup_{\lambda \in \Lambda} \overline{A_{\lambda}}^f$

$$\Rightarrow \overline{\bigcup_{\lambda \in \Lambda} A_{\lambda}}^f = \bigcup_{\lambda \in \Lambda} \overline{A_{\lambda}}^f \subseteq U \Rightarrow \overline{\bigcup_{\lambda \in \Lambda} A_{\lambda}}^f \subseteq U \text{ is } \bigcup_{\lambda \in \Lambda} A_{\lambda} \text{ } sf\text{-Open.}$$

Proposition(3.8): Let A be nowhere dense in a $tp\text{-}s$ X then A is $f\text{-closed}$.

Proof: Assume that A is a subset of X so that A is nowhere dense then $(\bar{A})^\circ = \emptyset$ and $(\overline{\bar{A}})^\circ = \emptyset$, but $\emptyset \subseteq A \Rightarrow A$ is f -closed set.

Lemma(3.9): A subset A of a tp -s X is sf -open iff $\bar{A}^f \subseteq S\ker(A)$.

Proof:($\Rightarrow$) Let A be sf -open in X , then $\bar{A}^f \subseteq U$ when $A \subseteq U$ and U is sf -open in X , this emplace $\bar{A}^f \subseteq \cap \{U : A \subseteq U \text{ and } U \in s\text{-open}(X)\} = S\ker(A)$. ($\Leftarrow$) Conversely,

assume that $\bar{A}^f \subseteq S\ker(A) \Rightarrow \bar{A}^f \subseteq \cap \{U : A \subseteq U \text{ and } U \in s\text{-open}(X)\} \Rightarrow \bar{A}^f \subseteq U$ for all s -open set U in X .

Proposition(3.10): Let X be tp -s, then the arbitrary intersection of sf -open sets in X is sf -open set.

Proof: Let $\{A_\lambda : \lambda \in \Lambda\}$ be arbitrary collection of sf -open sets in a space X , and let $A = \cap_{\lambda \in \Lambda} A_\lambda$, let $x \in \bar{A}^f$ then by [Lemma(2.15)] we consider the following two cases.

Case 1: $\{x\}$ is nowhere dense

If $x \notin A$ then for some $\alpha \in \Lambda$ we have $x \notin A_\alpha$, as nowhere dense subset are feebly closed [Proposition(3.8)] there fore $x \notin S\ker(A)$. On the other hand by [Lemma(3.9)] A_α is sf -open, then, $x \in \bar{A}^f \subseteq \bar{A}_\alpha^f \subseteq S\ker(A)$

paradoxically $x \in A$ and hence $x \in S\ker(A) \Rightarrow \bar{A}^f \subseteq S\ker(A)$, A sf -open set.

Case 2: $\{x\}$ is preopen, Let $F = \overline{\{x\}}^\circ$ and $x \notin S\ker(A)$, $\exists$ semi closed set C containing X , so that $C \cap A = \emptyset$, $x \in F = \overline{\{x\}}^\circ \subseteq \bar{C}^\circ \subseteq C$. As F is an open set containing x and $x \in \bar{A}^f$ therefore, $F \cap A \neq \emptyset$ as $F \subseteq C \Rightarrow C \cap A = \emptyset$ paradoxically $x \in S\ker(A) \Rightarrow A$ sf -open set.

Proposition(3.11): If A is sf -open and B f -closed in a tp -s X then $A \cap B$ is sf -open in X .

Proof: Assume that $A \cap B \subseteq U$ where U is s -open set then $A \cap B \cap U^c = \emptyset$

$\Rightarrow A \cap (B \cap U^c) = \emptyset \Rightarrow A \subseteq B^c \cup U$, but $B^c \cup U$ s -open $\bar{A}^f \subseteq B^c \cup U$

$\Rightarrow \bar{A}^f \cap (B^c \cup U)^c = \emptyset \Rightarrow \bar{A}^f \cap B \subseteq U \Rightarrow \overline{\bar{A} \cap B}^f \subseteq U \Rightarrow A \cap B$ sf -open.

Proposition(3.12): Assume that X is a tp -s & $A \subseteq X$ then $\bar{A}^f$ is sf -open set.

Proof: Let $\bar{A}^f \subseteq G$ where G is s -open set, since $\overline{\bar{A}^f}^f = \bar{A}^f \subseteq U$

$\Rightarrow \overline{\bar{A}^f}^f \subseteq G \Rightarrow \bar{A}^f$ sf -open set.

Proposition(3.13): Assume that X is a tp -s & $A \subseteq X$ then $\bar{A}$ is sf -open set.

Proof: Let $\bar{A} \subseteq U$ where U is s -open set, since $\overline{\bar{A}^f} = \bar{A}^f = \bar{A}$ [Proposition(2.11)(9)]. Then $\overline{\bar{A}^f} = \bar{A} \Rightarrow \bar{A}^f \subseteq U$ where U is s -open $\Rightarrow \bar{A}$ is sf -open set.

Proposition(3.14): Assume that X is an tp - s & $A \subseteq X$, if A is s -closed and pre closed then A is sf -open set.

Proof: Let A is s -closed then $A^\circ = \bar{A}^\circ$, since A pre closed then $\bar{A}^\circ \subseteq A$, but $A^\circ = \bar{A}^\circ$ then $\bar{A}^\circ \subseteq A \Rightarrow A$ f -closed by using [Remark(3.3)] A is sf -open set.

Definition(3.15): Assume that X is a tp - s & $A \subseteq X$. Then the intersection of all sf -closed of X which containing A is named sf -closure of A and shortened by $\bar{A}^{sf}$, that means $\bar{A}^{sf} = \cap \{F : F \text{ is } sf\text{-closed in } X\}$.

Lemma(3.16): Assume that X is a tp - s & $A \subseteq X$. Then $x \in \bar{A}^{sf}$ iff for all sf -open set G and $x \in G$, $G \cap A \neq \emptyset$.

Proof:($\Rightarrow$) Assume that $x \notin \bar{A}^{sf}$ then $x \notin \cap \{F : F \text{ is } sf\text{-closed in } X\}$ and $A \subseteq F$, then $x \in [\cap F]^c$, $[\cap F]^c$ sf -open containing x . Hence $[\cap F]^c \cap A \subseteq [\cap F]^c \cap [\cap F] = \emptyset$. ($\Leftarrow$) Conversely, Suppose that $\exists$ sf -open set G so that $x \in G$, $G \cap A = \emptyset$ then $A \subseteq G^c$, G^c is sf -closed hence $x \notin \bar{A}^{sf}$.

Definition(3.17): Let X be a tp - s , $x \in X$ & $A \subseteq X$. The point x is called sf -limit point of A if each sf -open set containing U , contains a point of A distinct from x . We shall call the set of all sf -limit point of A the sf - derivative set of A and denoted by A'^{sf} . Therefore $x \in A'^{sf}$ if for every sf -open set U in X such $x \in U$ implies that $\cap (A - \{x\}) \neq \emptyset$.

Proposition(3.18): Let X be a tp - s and $A \subseteq B \subseteq X$. Then:

1. $\bar{A}^{sf} = A \cup A'^{sf}$.
2. A is an sf -closed set iff $A'^{sf} \subseteq A$.
3. $A'^{sf} \subseteq B'^{sf}$.

Proof: 1- By definition $A \subseteq \bar{A}^{sf}$ (1). Let $x \in A'^{sf} \Rightarrow x \notin A$. Then $\forall$ sf -open set U contained x , then $(U \cap A) - \{x\} \neq \emptyset$. Then $\forall$ sf -open set in U contained x , then $U \cap A \neq \emptyset$ by [Lemma(3.16)]. Then $x \in \bar{A}^{sf} \Rightarrow A'^{sf} \subseteq \bar{A}^{sf}$ (2). From (1) and (2) $A \cup A'^{sf} \subseteq \bar{A}^{sf}$.

Let $x \in \bar{A}^{sf}$. Since $A \subseteq \bar{A}^{sf}$ by definition and $\because x \in \bar{A}^{sf}$ Then either $x \in A$ or $x \notin A$. If $x \in A \Rightarrow x \in A \cup A'^{sf}$ and if $x \notin A$. Since $x \in \bar{A}^{sf} \Rightarrow \forall$ sf -open set U contained x , then $U \cap A \neq \emptyset$, Since $x \notin A$ then $(U \cap A) - \{x\} \neq \emptyset$. Then $x \in A'^{sf} \Rightarrow x \in A \cup A'^{sf}$ then $\bar{A}^{sf} \subseteq A \cup A'^{sf}$ then $\bar{A}^{sf} = A \cup A'^{sf}$.

2- ($\Rightarrow$) Let $A'^{sf} \subseteq A$. $\bar{A}^{sf} = A \cup A'^{sf} \subseteq A$, Since $A \subseteq \bar{A}^{sf}$ then $A = \bar{A}^{sf}$, then A is an sf -closed set.

($\Leftarrow$) Let A be sf -closed set. Thus $A = \bar{A}^{sf}$ from [proposition (3.18)(1)]. $A = A \cup A'^{sf}$ then $A'^{sf} \subseteq A$.

3- Let $A \subseteq B$ and let $x \in A'^{sf}$, $\forall U$ is sf -open set contained x then $(U \cap A) - \{x\} \neq \emptyset$. Since $A \subseteq B \Rightarrow (U \cap B) - \{x\} \neq \emptyset$. Then $x \in B'^{sf}$ then $A'^{sf} \subseteq B'^{sf}$.

Remark(3.19): Assume that X is a tp -s & $A \subseteq X$, then $\bar{A}^{sf}$ is smallest sf -closed set containing A .

proof: Suppose that B is sf -closed set contend such that $A \subseteq B$ since $\bar{A}^{sf} = A \cup A'^{sf}$. And $\bar{A}^{sf} \subseteq \bar{B}^{sf}$, $A \subseteq B$, then $\bar{A}^{sf} = A \cup A'^{sf} \subseteq A \cup B'^{sf} \subseteq B$, then $\bar{A}^{sf} \subseteq B$ therefore $\bar{A}^{sf}$ is smallest sf -closed set contained A .

Proposition(3.20): Let X be a tp -s & A, B are subset of X with B sf -open set. If $x \in B$ and $B \cap A = \emptyset$ then $x \notin \bar{A}^{sf}$.

proof: Suppose $x \in \bar{A}^{sf}$, then either $x \in A$ or $x \in A'^{sf}$. If $x \in A$, then $B \cap A \neq \emptyset$ which contradicts the assumption and if $x \in A'^{sf}$ and $x \notin A$, then $(B \cap A) - \{x\} \neq \emptyset$ for every sf -open G in X containing x and hence $G \cap A \neq \emptyset$ which is a contradiction since B is sf -open set containing x and $B \cap A = \emptyset$ and hence $x \notin \bar{A}^{sf}$.

Definition(3.21): Assume that X is a tp -s & $B \subseteq X$. An sf -neighborhood of B is any subset of X which contains an sf -open set containing B . The sf -neighborhood of a subset $\{x\}$ is also called sf -neighborhood of the point x .

Definition(3.22): Assume that A is a subset of a tp -s X . For each $x \in X$, then x is said to be sf -boundary point of A if each sf -neighborhood U_x of x , we have $U_x \cap A \neq \emptyset$ and $U_x \cap A^c \neq \emptyset$. The set of all sf -boundary point of A is denoted by $b_{sf}(A)$.

Proposition(3.23): Assume that X is a tp -s and $A, B \subseteq X$, then

1. A is an sf -closed set $\Leftrightarrow A = \bar{A}^{sf}$.
2. $\bar{A}^{sf} \subseteq \bar{A}$.
3. $\bar{A}^{sf} = \overline{\bar{A}^{sf}}^{sf}$.
4. If $A \subseteq B$ then $\bar{A}^{sf} \subseteq \bar{B}^{sf}$.

proof: 1- ($\Rightarrow$) Let A is an sf -closed set. Since $A \subseteq \bar{A}^{sf}$. Then $\bar{A}^{sf} \subseteq A$ (since $\bar{A}^{sf}$ is the smallest sf -closed set containing A), then $A = \bar{A}^{sf}$.

($\Leftarrow$) Let $\overline{A}^{sf} = A$. Then $\overline{A}^{sf}$ is an sf -closed set as $A = \overline{A}^{sf} \Rightarrow A$ is a sf -closed set.

2- Let $x \in \overline{A}^{sf}$ and A is a sf -closed set, then $A = \overline{A}^{sf} \Rightarrow x \in A \subseteq \overline{A}$. Then $x \in \overline{A}$. Therefore $\overline{A}^{sf} \subseteq \overline{A}$.

3- Since $\overline{A}^{sf}$ is sf -closed set, then $\overline{A}^{sf} = \overline{\overline{A}^{sf}}^{sf}$ by (2).

4- Let $A \subseteq B$ and $B \subseteq \overline{B}^{sf}$, then $A \subseteq \overline{B}^{sf} \Rightarrow \overline{B}^{sf}$ is a sf -closed set containing A . Since $\overline{A}^{sf}$ is smallest sf -closed set containing A . Then $\overline{A}^{sf} \subseteq \overline{B}^{sf}$.

Definition(3.24): Assume that X is tp -s and $A \subseteq X$. The union of all sf -open sets of X contained in A is named sf -Interior of A , shortened by $A^{\circ sf}$ or sf - $In_{\tau}(A)$, that means sf - $In_{\tau}(A) = \bigcup \{B : B \text{ is } sf\text{-open in } X \text{ and } B \subseteq A\}$.

Proposition(3.25): Assume that X is tp -s & $A \subseteq X$. Then $\overline{A}^{sf} = (A^{c \circ sf})^c$.

Proof: Since $A \subseteq \overline{A}^{sf} \Rightarrow \overline{A}^{sf c} \subseteq A^c \Rightarrow \overline{A}^{sf c \circ sf} \subseteq A^{c \circ sf} \Rightarrow \overline{A}^{sf c} \subseteq A^{c \circ sf} \Rightarrow A^{c \circ sf c} \subseteq \overline{A}^{sf} \dots \dots (1)$.

Since $A^{c \circ sf} \subseteq A^c \Rightarrow A \subseteq A^{c \circ sf c} \Rightarrow \overline{A}^{sf} \subseteq \overline{A^{c \circ sf c}}^{sf} = A^{c \circ sf c} \dots \dots (2)$. From (1) and (2) we get $\overline{A}^{sf} = (A^{c \circ sf})^c$.

Proposition(3.26): Assume that X is tp -s & $A \subseteq X$. Then $x \in A^{\circ sf}$ iff there is an sf -open set U containing x so that $x \in U \subseteq A$.

Proof: Assume that $x \in A^{\circ sf} \Leftrightarrow x \in \bigcup \{U : U \subseteq A \text{ such that } U \text{ is } sf\text{-open in } X\} \Leftrightarrow \exists U \text{ is } sf\text{-open in } X \text{ so that } x \in U \subseteq A$.

Proposition(3.27): Assume that X tp -s & $A \subseteq B \subseteq X$, then:

1. $A^{\circ sf}$ is an sf -open set.
2. A is an sf -open set iff $A = A^{\circ sf}$.
3. $A^{\circ sf} = A^{\circ sf \circ sf}$.
4. If $A \subseteq B$ then $A^{\circ sf} \subseteq B^{\circ sf}$.

Proof: 1- $A^{\circ sf} = \bigcup \{B : B \text{ is } sf\text{-open and } B \subseteq A\}$, by [proposition(3. 7)]. Then $A^{\circ sf}$ is an sf -open set.

2- ($\Rightarrow$) Let A be an sf -open set from definition $A^{\circ sf} \subseteq A$, $A^{\circ sf} = \bigcup \{U : U \subseteq A, U \text{ is an } sf\text{-open set in } X\}$. Since A is sf -open set in X . Then $A \subseteq A^{\circ sf} \Rightarrow A = A^{\circ sf}$.

($\Leftarrow$) Let $A = A^{\circ sf}$, since $A^{\circ sf}$ is the union sf -open sets and since $A^{\circ sf} = A \Rightarrow A$ is a sf -open set.

3- Let $A^{\circ sf} = \cup\{B: B \text{ is an } sf\text{-open set in } X \text{ and } B \subseteq A\}$. $A^{\circ sf \circ sf} = \cup\{B: B \text{ is an } sf\text{-open set in } X \text{ and } B \subseteq A\}$ By (2) $A = A^{\circ sf}$. Then $A^{\circ sf} = A^{\circ sf \circ sf}$.

4- Let $A \subseteq B$ & $x \in A^{\circ sf}$. Then $\exists sf\text{-open } U \text{ in } X \text{ such that } x \in U \subseteq A$. Since $A \subseteq B$. Then $\exists sf\text{-open } U \text{ in } X \text{ such that } x \in U \subseteq A \subseteq B$. $x \in U \subseteq B \Rightarrow x \in B^{\circ sf}$. Then $A^{\circ sf} \subseteq B^{\circ sf}$.

Proposition(3.28): Assume that X is a $tp\text{-}s$ & $A \subseteq X$. Then:

$$1. b_{sf}(A) = \overline{A}^{sf} \cap \overline{A^c}^{sf}.$$

$$2. A^{\circ sf} = A - b_{sf}(A).$$

$$3. \overline{A}^{sf} = A \cup b_{sf}(A).$$

Proof: Clear

Proposition(3.29): Assume that X is a $tp\text{-}s$ & $A \subseteq X$. Then:

$$1- \overline{A}^{sf} = A^{\circ sf} \cup b_{sf}(A).$$

$$2- A \text{ is an } sf\text{-open set} \Leftrightarrow b_{sf}(A) \subseteq A^c.$$

$$3- (A)^{\circ sf} = (\overline{A^c}^{sf})^c.$$

Proof: Clear

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