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On s -g-cocompact open set and Continuity

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ABSTRACT

Throughout this paper by a space we mean a supra topological space, we have studied some of properties to new set is called supra generalized-cocompact open set (s -g-coc-open set) and find the relation with other sets and our concluded a new class of the function called s -g-coc-continuous, s -g-coc'-continuous . We shall provide some properties of these concepts and it will explain the relationship among them and some results on this subjects are proved. Throughout this work , and new concept have been illustrated including , s -g-coc-compact space .

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1. Introduction

In 1983, by dropping the finite intersection condition of topological spaces, Mashhour et al.[3] came up with an idea of supra topological spaces and it deals with groups called supra open sets. They introduced the notions of (irresolute) S^* -continuous maps and investigated some of their properties. Ravi et al. [2] introduced the idea of supra g-closed sets and obtained other properties of supra g-closed sets. [1] Al-shami introduced some studies on supra topology and the definition of supra open sets and their properties and presented some types of supra compact space . in [4],[5],[6] B. Meera Devi, D.K. Nathan R. Selvarathi. O. R. SAYED,] B.Meera Devi , respectively, introduced new class of functions in supra topological spaces, namely supra δg^* -continuous functions. In [7] S. Al Ghour and S. Samarah introduced coc-open sets in topological spaces.

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2. Preliminaries

Definition(2.1) [3] If X belongs to μ and μ is closed under arbitrary union, a subcollection μ of 2^X is considered a supra topology on X . That μ elements is referred to as a supra open sets ($\mathfrak{s}$ -open) of (X, μ) and its complement is referred to as supra closed sets ($\mathfrak{s}$ -closed). The supra closure of A (denoted by $\overline{A}^{\mathfrak{s}}$) is the intersection of all supra closed sets containing A and the supra interior of A (denoted by $A^{\circ\mathfrak{s}}$) is the union of all supra open sets contained in A . The supra relative topology μ_Y on Y is defined as $\mu_Y = \{ Y \cap G : G \in \mu \}$. A pair (Y, μ_Y) is called supra subspace of (X, μ) .

Definition(2.2) [2] Let (X, μ) be a supra topological space. A subset A of X is called supra $\mathfrak{g}$ -closed ($\mathfrak{s}$ - $\mathfrak{g}$ -closed) if $\overline{A}^{\mathfrak{s}} \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ) . And A is called supra $\mathfrak{g}$ -open ($\mathfrak{s}$ - $\mathfrak{g}$ -open), if A^c is supra $\mathfrak{g}$ -closed.

Remark(2.3) [2] Every $\mathfrak{s}$ -closed set implies $\mathfrak{s}$ - $\mathfrak{g}$ -closed set, $\mathfrak{s}$ - $\mathfrak{g}$ -closed set need not imply $\mathfrak{s}$ -closed sets, $\mathfrak{s}$ -open set implies $\mathfrak{s}$ - $\mathfrak{g}$ -open and $\mathfrak{s}$ - $\mathfrak{g}$ -open must not imply $\mathfrak{s}$ -open set.

Theorem(2.4) [2] A subset of supra topological space in (X, μ) is $\mathfrak{s}$ - $\mathfrak{g}$ -open set if and only if $B \subseteq A^{\circ\mathfrak{s}}$ whenever B is $\mathfrak{s}$ -closed and $B \subseteq A$.

Definition (2.5) [1] A supra topological spaces (X, μ) is called supra compact ($\mathfrak{s}$ -compact for short) provided that every supra open cover of X has a finite subcover.

Definition (2.6) [1]. A collection $\{G_i : i \in I\}$ of supra open sets in a supra topological spaces (X, μ) is called supra open cover of a subset E of X provided that $E \subseteq \bigcup \{G_i : i \in I\}$.

3. On supra generalized co-compact-open set and supra generalized co-compact -continuous functions

In this section, we introduce the definition of $\mathfrak{s}$ - $\mathfrak{g}$ -coc-open set, $\mathfrak{s}$ - $\mathfrak{g}$ -coc-closed set, $\mathfrak{s}$ - $\mathfrak{g}$ -coc-continuous, $\mathfrak{s}$ - $\mathfrak{g}$ -coc'-continuous, remarks and propositions about this concept.

Definition (3.1) A subset A of a space (X, μ) is called supra generalized cocompact open set ($\mathfrak{s}$ - $\mathfrak{g}$ -coc-open set) if for every $x \in A$ there exists $\mathfrak{s}$ - $\mathfrak{g}$ -open set $U \subseteq X$ and $\mathfrak{s}$ -compact subset K such that $x \in U - K \subseteq A$, the complement of $\mathfrak{s}$ - $\mathfrak{g}$ -coc-open set is called $\mathfrak{s}$ - $\mathfrak{g}$ -coc-closed set. The family of all $\mathfrak{s}$ - $\mathfrak{g}$ -coc-open subsets of a space (X, μ) will be denoted by $\mu^{\mathfrak{gk}}$.

Example(3.4) Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, X, \{a, b\}, \{a, c\}\}$. Then $\mu^{\mathfrak{gk}} = p(X)$

Proposition (3.5)

i- Every $\mathfrak{s}$ -open set is $\mathfrak{s}$ - $\mathfrak{g}$ -coc-open set.

iii- Every $\mathfrak{s}$ - $\mathfrak{g}$ -open set is $\mathfrak{s}$ - $\mathfrak{g}$ -coc-open set.

proof

i. Let A $\mathfrak{s}$ -open set Then A $\mathfrak{s}$ - $\mathfrak{g}$ -open and let K $\mathfrak{s}$ -compact ($\emptyset$). Then for all $x \in A$ we have $x \in U - K \subseteq A$

ii. Let A $\mathfrak{s}$ -g-open and K $\mathfrak{s}$ -compact ($\emptyset$). Then for all $x \in A$ we have $x \in U - K \subseteq A$

but the converses of i and ii are not true for the following example :-

Example (3.6)

Let $X = \mathbb{N}$. $\mu = \{\emptyset, X, X - \{1\}\}$ is supra topological space. It is clear that $\{1\}$ $\mathfrak{s}$ -g-coc-open set but not $\mathfrak{s}$ -open and not $\mathfrak{s}$ -g-open set .

Remark (3.7) :- Every $\mathfrak{s}$ -g-closed is $\mathfrak{s}$ -g-coc-closed but the converse is not true as in the example (3.6) .It is clear that $\{1\}^c$ $\mathfrak{s}$ -g-coc-closed set but $\{1\}^c$ not $\mathfrak{s}$ -g-closed set .

Proposition (3.8) :- The union of $\mathfrak{s}$ -g-coc-open is $\mathfrak{s}$ -g-coc-open set

Proof

let $\{A_\kappa : \kappa \in \Lambda\}$ $\mathfrak{s}$ -g-coc-open sets . let $x \in A_\kappa$.Then for some $\kappa \in \Lambda$.Then there exists U_κ $\mathfrak{s}$ -g-open set and K_κ $\mathfrak{s}$ -compact such that $x \in U_\kappa - K_\kappa \subseteq A_\kappa \subseteq \bigcup A_\kappa$ then $x \in U_\kappa - K_\kappa \subseteq \bigcup A_\kappa$,then $\bigcup A_\kappa$ $\mathfrak{s}$ -g-coc-open set .

Theorem (3.9) :- μ^{gk} forms supra topology on X .

Proof.

By the definition one has directly that $\emptyset \in \mu^{\text{gk}}$. To see that $X \in \mu^{\text{gk}}$, let $x \in X$, take $U = X$ and $K = \emptyset$. Then $x \in U - K \subseteq X$.

Let $\{U_\alpha : \alpha \in \Delta\}$ be a collection of $\mathfrak{s}$ -g-coc-open subsets of (X, μ) and $x \in \bigcup_{\alpha \in \Delta} U_\alpha$. Then there exists $\alpha_0 \in \Delta$ such that $x \in U_{\alpha_0}$. Since U_{α_0} is $\mathfrak{s}$ -g-coc-open, then there exists an $\mathfrak{s}$ -g-open set V and $\mathfrak{s}$ -compact subset K , such that $x \in V - K \subseteq U_{\alpha_0}$. Therefore, we have $x \in V - K \subseteq U_{\alpha_0} \subseteq \bigcup_{\alpha \in \Delta} U_\alpha$. Hence, $\bigcup_{\alpha \in \Delta} U_\alpha$ is $\mathfrak{s}$ -g-coc-open.

The following diagram shows the relation between types of $\mathfrak{s}$ -coc-open sets

Definition (3.10) Let X be a space and $A \subseteq X$. The intersection of all $\mathfrak{s}$ -g-coc-closed sets of X containing A is called the $\mathfrak{s}$ -g-coc-closure of A defined by .

$$\overline{A}^{\mathfrak{s}-g-coc} = \bigcap \{B : B \text{ } \mathfrak{s}\text{-g-coc-closed in } X \text{ and } A \subseteq B\}$$

Proposition (3.11) Let X be a space and $A \subseteq X$, then $x \in \overline{A}^{\mathfrak{s}-g-coc}$ if and only if for each $\mathfrak{s}$ -g-coc-open in X contained point x we have $U \cap A \neq \emptyset$.

Proof :

Asume that $x \in \overline{A}^{\mathfrak{s}-g-coc}$ and let U $\mathfrak{s}$ -g-coc-open in X , such that $x \in U$, and suppose $U \cap A = \emptyset$, then $A \subseteq U^c$ since U is $\mathfrak{s}$ -g-coc-open set in X , $x \in U$, then U^c is $\mathfrak{s}$ -g-coc-closed set in X , $x \notin U^c$, and $\overline{A}^{\mathfrak{s}-g-coc}$ is smallest $\mathfrak{s}$ -g-coc-

closed set contain A, then $\overline{A}^{\overline{s-g-coc}} \subseteq U^c$, since $U \cap U^c = \emptyset$ $x \in U$ then $x \notin U^c$, then $x \notin \overline{A}^{\overline{s-g-coc}}$, and this contradiction, thus $\forall$ s -g-coc-open U containing x we have $U \cap A \neq \emptyset$.

Conversely; Let U is s -g-coc-open set in X , such that $x \in U$ and $U \cap A \neq \emptyset$. To prove $x \in \overline{A}^{\overline{s-g-coc}}$, let $x \notin \overline{A}^{\overline{s-g-coc}}$, then $x \in (\overline{A}^{\overline{s-g-coc}})^c$, since $\overline{A}^{\overline{s-g-coc}}$ is s -g-coc-closed, then $(\overline{A}^{\overline{s-g-coc}})^c$ is s -g-coc-open set in X , and $\overline{A}^{\overline{s-g-coc}} \cap (\overline{A}^{\overline{s-g-coc}})^c = \emptyset$, which is contradiction since $A \cap U \neq \emptyset$, $\forall$ U is s -g-coc-open set in X , such that $x \in U$.

Proposition (3.12)

Let X be a space and $A \subseteq B \subseteq X$ then

1. $\overline{A}^{\overline{s-g-coc}}$ is s -g-coc-closed set
2. A is s -g-coc-closed if and only if $A = \overline{A}^{\overline{s-g-coc}}$
3. $\overline{A}^{\overline{s-g-coc}} = \overline{\overline{A}^{\overline{s-g-coc}}}$
4. If $A \subseteq B$ then $\overline{A}^{\overline{s-g-coc}} \subseteq \overline{B}^{\overline{s-g-coc}}$
5. $\overline{A}^{\overline{s-g-coc}} \subseteq \overline{A}^{\overline{s}}$

Proof :-

1. By Proposition (3.8).
2. Let A be s -g-coc-closed in X . Since $A \subseteq \overline{A}^{\overline{s-g-coc}}$ and $\overline{A}^{\overline{s-g-coc}}$ smallest s -g-coc-closed set containing A , then $\overline{A}^{\overline{s-g-coc}} \subseteq A$ then $A = \overline{A}^{\overline{s-g-coc}}$
conversely :- Let $A = \overline{A}^{\overline{s-g-coc}}$. Since $\overline{A}^{\overline{s-g-coc}}$ is s -g-coc-closed then A is s -g-coc-closed.
3. From (1) and (2)
4. Let $A \subseteq B$. Since $B \subseteq \overline{B}^{\overline{s-g-coc}}$ then $A \subseteq \overline{B}^{\overline{s-g-coc}}$. Since $\overline{B}^{\overline{s-g-coc}}$ smallest s -g-coc-closed set containing A then $\overline{A}^{\overline{s-g-coc}} \subseteq \overline{B}^{\overline{s-g-coc}}$ (since $\overline{A}^{\overline{s-g-coc}}$ smallest s -g-coc-closed set containing A).
5. Let $x \in \overline{A}^{\overline{s-g-coc}}$ then for all s -g-coc-open set U , such that $x \in U$ we have $U \cap A \neq \emptyset$. Then for all s -open set U , such that $x \in U$, we have $A \cap U \neq \emptyset$ by proposition (3.11). Then $x \in \overline{A}^{\overline{s}}$.

Definition (3.13) Let X be a space and $A \subseteq X$. The union of all s -g-coc-open sets of X containing in A is called s -g-coc-Interior of A denoted by $A^{\circ s-g-coc}$ or s -g-coc-In (A)

$$A^{\circ s-g-coc} = \bigcup \{B : B \text{ } s\text{-g-coc-open in } X \text{ and } B \subseteq A\}$$

Proposition (3.14) Let X be a space and $A \subseteq X$, then $x \in A^{\circ s-g-coc}$ if and only if there exists s -g-coc-open set V containing x such that $x \in V \subseteq A$.

Proof :

Let $x \in A^{\circ s-g-coc}$ then $x \in \bigcup V$ such that V s -g-coc-open set and $x \in V \subseteq A$.

Conversely; Let there exists V s -g-coc-open set, such that $x \in V \subseteq A$ then $x \in \bigcup V$, $V \subseteq A$ and V s -g-coc-open set, then $x \in A^{\circ s-g-coc}$.

Proposition (3.15)

Let X be a space and $A \subseteq B \subseteq X$, then .

1. $A^{\circ s-g-coc}$ is s -g-coc-open set .
2. A is s -g-coc-open if and only if $A = A^{\circ s-g-coc}$.
3. $A^{\circ s} \subseteq A^{\circ s-g-coc}$.

$$4. A^{\circ s-g-coc} = (A^{\circ s-g-coc})^{\circ s-g-coc}.$$

$$5. \text{ if } A \subseteq B \text{ then } A^{\circ s-g-coc} \subseteq B^{\circ s-g-coc}.$$

Proof :-

1. By Proposition (3.8) .

2. let A is s -g-coc-open ,since $A^{\circ s-g-coc}$ largest s -g-coc-open containing A ,then $A^{\circ s-g-coc} \subseteq A$ and since $A \subseteq A^{\circ s-g-coc}$, then $A = A^{\circ s-g-coc}$.

Conversely; Let $A = A^{\circ s-g-coc}$,since $A^{\circ s-g-coc}$ is s -g-coc-open ,then A is s -g-coc-open .

3. Let $x \in A^{\circ s}$ then there exists U s -open set such that $x \in U \subseteq A$ then U s -g-coc-open set then U s -g-coc-open set such that $x \in U \subseteq A$ thus $x \in A^{\circ s-g-coc}$

4. from (1) and (2).

5. Let $x \in A^{\circ s-g-coc}$ then there exists V s -g-coc-open set such that $x \in V \subseteq A$ by proposition(3.19), since $A \subseteq B$ then $x \in V \subseteq B$.Then $x \in B^{\circ s-g-coc}$ by proposition(3.14),. Thus $A^{\circ s-g-coc} \subseteq B^{\circ s-g-coc}$.

Definition (3.16)[3] Let $f: X \rightarrow Y$ be a function of space X into space Y . Then f is called supra-irresolute (s -continuous) function if $f^{-1}(A)$ is s -open set in X for every s -open set A in Y

Definition (3.17) Let $f: X \rightarrow Y$ be a function of a space X into a space Y . f is called s -g-coc-continuous function if $f^{-1}(A)$ is s -g-coc-open set in X for every s -open set in Y .

Proposition (3.18) Every s -continuous is s -g-coc-continuous

Proof

Let $f: X \rightarrow Y$ be s -continuous function and A s -open set in Y . Thus $f^{-1}(A)$ is s -open set in X . Then $f^{-1}(A)$ is s -g-coc-open set in X .Then f is s -g-coc-continuous

But the convers not true in general for example

Example (3.19)

Let $X = \mathbb{N}$, $\mu = \{\emptyset, X, \{2\}\}$ supra topology on X . $Y = \{a, b, c\}$. $\mu = \{\emptyset, Y, \{a\}\}$ supra topology on Y and $f: X \rightarrow Y$ defined by $f(x) = \begin{cases} a & \text{if } x \in \{1, 3\} \\ b & \text{if } x \notin \{1, 3\} \end{cases}$. Then f is s -g-coc-continuous But not s -continuous

Remark (3.20)

Let $f: X \rightarrow Y$ be a function of a space X into a space Y then

- The constant function is s -g-coc-continuous
- If (X, μ) supra discrete topology then f s -g-coc-continuous
- If X finite set and μ any topology on X then f s -g-coc-continuous
- If (Y, μ') indiscrete topology then f s -g-coc-continuous

Proposition (3.21)

Let $f: X \rightarrow Y$ be a function of a space X into a space Y then the following statements are equivalent :-

- f s -g-coc-continuous function .
- $f^{-1}(A^{\circ s}) \subseteq (f^{-1}(A))^{\circ s-g-coc}$ for every set A of Y .
- $f^{-1}(A)$ s -g-coc-closed set in X for every s -closed set A in Y .

4. $f(\bar{A}^{\mathfrak{s}-g-coc}) \subseteq \overline{f(A)}^{\mathfrak{s}}$ for every set A of X .

5. $\overline{f^{-1}(A)}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(\bar{A}^{\mathfrak{s}})$ for every set A of Y .

proof:

1 $\rightarrow$ 2

Let $A \subseteq Y$, since $A^{\mathfrak{s}}$ is $\mathfrak{s}$ -open set in Y . Then $f^{-1}(A^{\mathfrak{s}})$ is $\mathfrak{s}$ -g-coc-open set in X . Thus $f^{-1}(A^{\mathfrak{s}}) = (f^{-1}(A^{\mathfrak{s}}))^{\mathfrak{s}-g-coc} \subseteq (f^{-1}(A))^{\mathfrak{s}-g-coc}$. Hence $f^{-1}(A^{\mathfrak{s}}) \subseteq (f(A))^{\mathfrak{s}-g-coc}$.

2 $\rightarrow$ 3

Let $A \subseteq Y$ such that A is $\mathfrak{s}$ -closed set in Y . Then A^c is $\mathfrak{s}$ -open set then $A^c = (A^c)^{\mathfrak{s}}$ then $f^{-1}((A^c)^{\mathfrak{s}}) \subseteq (f^{-1}(A^c))^{\mathfrak{s}-g-coc}$. Therefore $f^{-1}(A^c) \subseteq (f^{-1}(A^c))^{\mathfrak{s}-g-coc}$ then $(f^{-1}(A))^c \subseteq (f^{-1}(A)^c)^{\mathfrak{s}-g-coc}$. Therefore $(f^{-1}(A))^c = (f^{-1}(A)^c)^{\mathfrak{s}-g-coc}$. Hence $(f^{-1}(A))^c$ is $\mathfrak{s}$ -g-coc-open set in X . Hence $f^{-1}(A)$ is $\mathfrak{s}$ -g-coc-closed set in X .

3 $\rightarrow$ 4

Let $A \subseteq X$. Then $\overline{f(A)}^{\mathfrak{s}}$ is $\mathfrak{s}$ -g-coc-closed set in Y . Then by (3) we have $f^{-1}(\overline{f(A)}^{\mathfrak{s}})$ is $\mathfrak{s}$ -g-coc-closed set in X containing A , thus $\bar{A}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(\overline{f(A)}^{\mathfrak{s}})$ (since $\bar{A}^{\mathfrak{s}-g-coc}$ is intersection of all $\mathfrak{s}$ -g-coc-closed sets in X containing A). Hence $f(\bar{A}^{\mathfrak{s}-g-coc}) \subseteq \overline{f(A)}^{\mathfrak{s}}$.

4 $\rightarrow$ 5

Let $A \subseteq Y$. Then $f^{-1}(A) \subseteq X$. Then by (4) we have $f(\overline{(f^{-1}(A))^{\mathfrak{s}-g-coc}}) \subseteq \overline{f(f^{-1}(A))^{\mathfrak{s}}}$. Hence $\overline{f^{-1}(A)}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(\bar{A}^{\mathfrak{s}})$.

5 $\rightarrow$ 1

Let B is $\mathfrak{s}$ -open set in Y then B^c is $\mathfrak{s}$ -closed. Then $B^c = \overline{B^c}^{\mathfrak{s}}$. Hence $\overline{f^{-1}(B^c)}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(\overline{B^c}^{\mathfrak{s}})$. Then $\overline{f^{-1}(B^c)}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(B^c)$. Then $f^{-1}(B^c) = (f^{-1}(B))^c$ is $\mathfrak{s}$ -g-coc-closed set in X . Therefore $f^{-1}(B)$ is $\mathfrak{s}$ -g-coc-open set in X . Thus f is $\mathfrak{s}$ -g-coc-continuous function.

Remark (3.22) From proposition (3.21) we have f is $\mathfrak{s}$ -g-coc-continuous if and only if, the inverse image of every $\mathfrak{s}$ -closed set in Y is $\mathfrak{s}$ -g-coc-closed set in X .

Definition (3.23) Let $f: X \rightarrow Y$ be a function of a space X into a space Y , then f is called $\mathfrak{s}$ -g-coc irresolute ($\mathfrak{s}$ -g-coc-continuous) function if $f^{-1}(A)$ is $\mathfrak{s}$ -g-coc-open set in X for every $\mathfrak{s}$ -g-coc-open set in Y .

Proposition (3.24) Every $\mathfrak{s}$ -g-coc-continuous function is $\mathfrak{s}$ -g-coc-continuous function

Proof:

Let $f: (X, \mu) \rightarrow (Y, \mu')$ be $\mathfrak{s}$ -g-coc-continuous and B is $\mathfrak{s}$ -open set in Y . Then B is $\mathfrak{s}$ -g-coc-open set. Since f is $\mathfrak{s}$ -g-coc-continuous then $f^{-1}(B)$ is $\mathfrak{s}$ -g-coc-open. Hence f is $\mathfrak{s}$ -g-coc-continuous function.

But the converse is not true in general for the following example.

Example(3.25): $f: X \rightarrow Y$ be function defined by $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{N} \\ 2 & \text{if } x = x_1 \\ 3 & \text{if } x \in \mathbb{N}_0 \end{cases}$ and $X = x_1 \cup \mathbb{N}$ ($x_1 \in \mathbb{R}$, $\mathbb{N}$ nature number, $\mu = \{ \emptyset, X, A: A \subseteq \mathbb{N} \}$, μ' supra indiscrete topology on $Y = \{1, 2, 3\}$, then the only $\mathfrak{s}$ -open sets in Y are Y and $\emptyset$, then $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(Y) = X$. Since Y and $\emptyset$ are $\mathfrak{s}$ -g-coc-open sets in Y then $f^{-1}(\emptyset), f^{-1}(Y)$ are $\mathfrak{s}$ -g-coc-

open in X . Then f $\mathfrak{s}$ -g-coc-continuous function. But $\{2\}$ $\mathfrak{s}$ -g-coc-open in Y and $f^{-1}(\{2\}) = \{x_1\}$ is not $\mathfrak{s}$ -g-coc-open set in X . $x_1 \in \{x_1\}$. There is no $\mathfrak{s}$ -g-open set U such that $x_1 \in U$, and K $\mathfrak{s}$ -compact such that $x_1 \in U - K \subseteq \{x_1\}$. Then f is not $\mathfrak{s}$ -g-coc'-continuous function.

Proposition (3.26) Let $f: X \rightarrow Y$ be $\mathfrak{s}$ -g-coc'-continuous then $f^{-1}(A)$ $\mathfrak{s}$ -g-coc-closed set in X for all A $\mathfrak{s}$ -g-coc-closed set in Y

proof

Let A is $\mathfrak{s}$ -g-coc-closed set in Y . Then A^c $\mathfrak{s}$ -g-coc-open in Y . Since f $\mathfrak{s}$ -g-coc'-continuous. Then $f^{-1}(A^c)$ is $\mathfrak{s}$ -g-coc-open in X by definition(2.10). Since $f^{-1}(A^c) = (f^{-1}(A))^c$. Then $(f^{-1}(A))^c$ $\mathfrak{s}$ -g-coc-open set in X . Therefore, $f^{-1}(A)$ $\mathfrak{s}$ -g-coc-closed set in X for all A $\mathfrak{s}$ -g-coc-closed set in Y .

Remark (3.27)

- i. $\mathfrak{s}$ -g-coc-continuous is need not to be $\mathfrak{s}$ -continuous function.
- ii. $\mathfrak{s}$ -continuous is need not to be $\mathfrak{s}$ -g-coc-continuous function.

But the converse is not true as the follow example

Examples (3.28)

i--

Let $f: X \rightarrow Y$. $X = \{1, 2, 3\}$. $\mu = \{\emptyset, X, \{3\}\}$ supra topology on X and $Y = \{a, b\}$. $\mu' = \{\emptyset, Y, \{a\}\}$ supra topology on Y . $f(1) = f(3) = b$. $f(2) = a$. It is clear that f is $\mathfrak{s}$ -g-coc-continuous but not $\mathfrak{s}$ -continuous.

ii- Let $f: X \rightarrow Y$ be function defined by $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{N} \\ 2 & \text{if } x = x_1 \text{ and } X = x_1 \cup \mathbb{N} \text{ (} x_1 \in \mathbb{R}, \mathbb{N} \text{ natural number)} \\ 3 & \text{if } x \in \mathbb{N}_0 \end{cases}$

$= \{\emptyset, X, A: A \subseteq \mathbb{N}\}$. μ' supra indiscrete topology on $Y = \{1, 2, 3\}$, then the only $\mathfrak{s}$ -open sets in Y are Y and $\emptyset$, then $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(Y) = X$. Since Y and $\emptyset$ $\mathfrak{s}$ -open sets in Y then $f^{-1}(\emptyset)$, $f^{-1}(Y)$ $\mathfrak{s}$ -open in X . Then f $\mathfrak{s}$ -continuous function. But $\{2\}$ $\mathfrak{s}$ -g-coc-open in Y and $f^{-1}(\{2\}) = \{x_1\}$ is not $\mathfrak{s}$ -g-coc-open set in X . $x_1 \in \{x_1\}$. There is no $\mathfrak{s}$ -g-open set U such that $x_1 \in U$, and K $\mathfrak{s}$ -compact such that $x_1 \in U - K \subseteq \{x_1\}$. Then f is not $\mathfrak{s}$ -g-coc'-continuous function.

Proposition (3.29)

Let $f: X \rightarrow Y$ be a function of space X into space Y then the following statements are equivalent.

- i. f is $\mathfrak{s}$ -g-coc-continuous.
- ii. $f(\overline{A}^{\mathfrak{s}-g-coc}) \subseteq \overline{f(A)}^{\mathfrak{s}-g-coc}$ for every set $A \subseteq X$.
- iii. $\overline{f^{-1}(B)}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(\overline{B}^{\mathfrak{s}-g-coc})$ for every set $B \subseteq Y$.

proof:

$i \rightarrow ii$

Let $A \subseteq X$ then $f(A) \subseteq Y$ and $\overline{f(A)}^{\mathfrak{s}-g-coc}$ $\mathfrak{s}$ -g-coc-closed set in Y . Since f is $\mathfrak{s}$ -g-coc-continuous. Then $f^{-1}(\overline{f(A)}^{\mathfrak{s}-g-coc})$ $\mathfrak{s}$ -g-coc-closed set in X by proposition(2.14). Since $f(A) \subseteq \overline{f(A)}^{\mathfrak{s}-g-coc}$ then $f^{-1}(f(A)) \subseteq f^{-1}(\overline{f(A)}^{\mathfrak{s}-g-coc})$. Then $A \subseteq f^{-1}(\overline{f(A)}^{\mathfrak{s}-g-coc})$. Since $f^{-1}(\overline{f(A)}^{\mathfrak{s}-g-coc})$ $\mathfrak{s}$ -g-coc-closed. Then $\overline{A}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(\overline{f(A)}^{\mathfrak{s}-g-coc})$ thus $f(\overline{A}^{\mathfrak{s}-g-coc}) \subseteq f(f^{-1}(\overline{f(A)}^{\mathfrak{s}-g-coc})) \subseteq \overline{f(A)}^{\mathfrak{s}-g-coc}$. Then $f(\overline{A}^{\mathfrak{s}-g-coc}) \subseteq \overline{f(A)}^{\mathfrak{s}-g-coc}$

$ii \rightarrow iii$

Let $(\overline{A}^{\mathfrak{s}-g-coc}) \subseteq \overline{f(A)}^{\mathfrak{s}-g-coc} \forall A \subseteq X$. $B \subseteq Y$, then $f^{-1}(B) \subseteq X$, $f(\overline{f^{-1}(B)}^{\mathfrak{s}-g-coc}) \subseteq \overline{f(f^{-1}(B))}^{\mathfrak{s}-g-coc}$, then $f(\overline{f^{-1}(B)}^{\mathfrak{s}-g-coc}) \subseteq \overline{B}^{\mathfrak{s}-g-coc}$. Hence $\overline{f^{-1}(B)}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(\overline{B}^{\mathfrak{s}-g-coc})$

iii $\rightarrow$ i

Let B be $\mathfrak{s}$ -g-coc-closed set in Y then $B = \overline{B}^{\mathfrak{s}-g-coc}$. Since $\overline{f^{-1}(B)}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(\overline{B}^{\mathfrak{s}-g-coc})$. Then $\overline{f^{-1}(B)}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(B)$. Since $f^{-1}(B) \subseteq \overline{f^{-1}(B)}^{\mathfrak{s}-g-coc}$. Therefore $\overline{f^{-1}(B)}^{\mathfrak{s}-g-coc} = f^{-1}(B)$. Therefore $f^{-1}(B)$ $\mathfrak{s}$ -g-coc-closed in X . Hence f is $\mathfrak{s}-g-coc$ -continuous.

Proposition (3.30) Let $f: X \rightarrow Y$ be a function of space X into space Y then f is $\mathfrak{s}-g-coc$ -continuous function if and only if the inverse image of every $\mathfrak{s}$ -g-coc-closed in Y is $\mathfrak{s}$ -g-coc-closed set in X

proof:

Let f be $\mathfrak{s}-g-coc$ -continuous, let B be $\mathfrak{s}$ -g-coc-closed set in Y . Then B^c $\mathfrak{s}$ -g-coc-open in Y . Since f $\mathfrak{s}-g-coc$ -continuous, then $f^{-1}(B^c)$ $\mathfrak{s}$ -g-coc-open in X . $f^{-1}(B^c) = f^{-1}(Y - B) = f^{-1}(Y) - f^{-1}(B) = X - f^{-1}(B) = (f^{-1}(B))^c$. Then $(f^{-1}(B))^c$ $\mathfrak{s}$ -g-coc-open in X . Hence $f^{-1}(B)$ $\mathfrak{s}$ -g-coc-closed in X .

Conversely:

Let M be $\mathfrak{s}$ -g-coc-open in Y , then M^c is $\mathfrak{s}$ -g-coc-closed in Y . Then $f^{-1}(M^c)$ $\mathfrak{s}$ -g-coc-closed in X , since $f^{-1}(M^c) = f^{-1}(Y - M) = f^{-1}(Y) - f^{-1}(M) = X - f^{-1}(M) = (f^{-1}(M))^c$. Therefore, $f^{-1}(M^c) = (f^{-1}(M))^c$. Then $(f^{-1}(M))^c$ $\mathfrak{s}$ -g-coc-closed in X hence $f^{-1}(M)$ $\mathfrak{s}$ -g-coc-open set in X then f is $\mathfrak{s}-g-coc$ -continuous.

The following diagram shows the relation between types of $\mathfrak{s}$ -coc-continuous functions

4. $\mathfrak{s}$ -g-Coc-compact space

In this section, we introduce the concept of $\mathfrak{s}$ -g-Coc-compact space and give some important generalization on this concept.

Definition (4.1): Let X be a supra space. A family F of subset of X is called $\mathfrak{s}$ -g-coc-open cover of X if F covers X and F is sub family of μ^{gk} .

Definition (4.2): A space X is said to be $\mathfrak{s}-g-coc$ -compact if every $\mathfrak{s}$ -g-coc-open cover of X has finite sub cover.

Example (4.3):

- i. Every finite subset of a space X is an $\mathfrak{s}$ -g-coc-compact.
- ii. The indiscrete space is $\mathfrak{s}$ -g-coc-compact space.

Remark (4.4): It is clear that every $\mathfrak{s}$ -g-coc-compact space is $\mathfrak{s}$ -compact but the converse is not true in general as the following example shows

Example (4.5): Let $X = \mathbb{R}$ the set of real numbers with τ is indiscrete topology, the coc-open set is $\{A: A \subseteq X\}$. Then X is $\mathfrak{s}$ -compact space but not $\mathfrak{s}$ -g-coc-compact. It is clear that, a space (X, μ) is $\mathfrak{s}$ -g-coc-compact iff the space (X, μ^{gk}) is $\mathfrak{s}$ -compact.

Theorem (4.6):

Let $f: X \rightarrow Y$ be an onto $\mathfrak{s}$ -g-coc-continuous function. If X is $\mathfrak{s}$ -g-coc-compact then Y is $\mathfrak{s}$ -compact.

Proof:

Let $\{V_\alpha : \alpha \in \Lambda\}$ be an $\mathfrak{s}$ -open cover of Y then $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ is an $\mathfrak{s}$ -g-coc-open cover of X , since X is $\mathfrak{s}$ -g-coc-compact. Then X has finite sub cover say $\{f^{-1}(V_{\alpha_i}) : i = 1, 2, \dots, n\}$ and $V_{\alpha_i} \in \{V_\alpha : \alpha \in \Lambda\}$. Hence $\{V_{\alpha_i} : i = 1, 2, \dots, n\}$ is a finite sub cover of Y . Then Y is $\mathfrak{s}$ -compact.

Theorem (4.7):

Let $f: X \rightarrow Y$ be an onto $\mathfrak{s}$ -g-coc'-continuous function. If X is $\mathfrak{s}$ -g-coc-compact then Y is $\mathfrak{s}$ -g-coc-compact.

Proof:

Let $\{V_\alpha : \alpha \in \Lambda\}$ be an $\mathfrak{s}$ -g-coc-open cover of Y then $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ is an $\mathfrak{s}$ -g-coc-open cover of X , since X is $\mathfrak{s}$ -g-coc-compact. Then X has finite sub cover say $\{f^{-1}(V_{\alpha_i}) : i = 1, 2, \dots, n\}$ and $V_{\alpha_i} \in \{V_\alpha : \alpha \in \Lambda\}$. Hence $\{V_{\alpha_i} : i = 1, 2, \dots, n\}$ is a finite sub cover of Y . Then Y is $\mathfrak{s}$ -g-coc-compact.

Proposition (4.8):

For any space X the following statement are equivalent:

- i. X is $\mathfrak{s}$ -g-coc-compact
- ii. Every family of $\mathfrak{s}$ -g-coc-closed sets $\{F_\alpha : \alpha \in \Lambda\}$ of X such that $\bigcap_{\alpha \in \Lambda} F_\alpha = \phi$, then there exist a finite subset $\Lambda_o \subseteq \Lambda$ such that $\bigcap_{\alpha \in \Lambda_o} F_\alpha = \phi$.

Proof:

(i) $\rightarrow$ (ii) Assume that X is $\mathfrak{s}$ -g-coc-compact, let $\{F_\alpha : \alpha \in \Lambda\}$ be a family of $\mathfrak{s}$ -g-coc-closed subset of X such that $\bigcap_{\alpha \in \Lambda} F_\alpha = \phi$. Then the family $\{X - F_\alpha : \alpha \in \Lambda\}$ is $\mathfrak{s}$ -g-coc-open cover of the $\mathfrak{s}$ -g-coc-compact (X, μ) there exist a finite subset Λ_o of Λ such that $X = \bigcup \{X - F_\alpha : \alpha \in \Lambda_o\}$ therefore $\phi = X - \bigcup \{X - F_\alpha : \alpha \in \Lambda_o\} = \bigcap \{X - (X - F_\alpha) : \alpha \in \Lambda_o\} = \bigcap \{F_\alpha : \alpha \in \Lambda_o\}$

(ii) $\rightarrow$ (i) Let $U = \{U_\alpha : \alpha \in \Lambda\}$ be an $\mathfrak{s}$ -g-coc-open cover of the space (X, τ) . Then $X - \{U_\alpha : \alpha \in \Lambda\}$ is a family of $\mathfrak{s}$ -g-coc-closed subset of (X, μ) with $\bigcap \{X - U_\alpha : \alpha \in \Lambda\} = \phi$ by assumption there exists a finite subset Λ_o of Λ such that $\bigcap \{X - U_\alpha : \alpha \in \Lambda_o\} = \phi$ so $X = X - \bigcap \{X - U_\alpha : \alpha \in \Lambda_o\} = \bigcup \{U_\alpha : \alpha \in \Lambda_o\}$. Hence X is $\mathfrak{s}$ -g-coc-compact.

Definition (4.9):

A subset B of a space X is said to be $\mathfrak{s}$ -g-coc-compact relative to X if for every cover of B by $\mathfrak{s}$ -g-coc-open sets of X has finite sub cover of B . The sub set B is $\mathfrak{s}$ -g-coc-compact iff it is $\mathfrak{s}$ -g-coc-compact as a sub space.

Proposition (4.10): If X is a space such that every $\mathfrak{s}$ -g-coc-open subset of X is $\mathfrak{s}$ -g-coc-compact relative to X , then every subset is $\mathfrak{s}$ -g-coc-compact relative to X .

Proof:

Let B be an arbitrary subset of X and let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of B by $\mathfrak{s}$ -g-coc-open sets of X . Then the family $\{U_\alpha : \alpha \in \Lambda\}$ is a $\mathfrak{s}$ -g-coc-open cover of the $\mathfrak{s}$ -g-coc-open set $\bigcup \{U_\alpha : \alpha \in \Lambda\}$ Hence by hypothesis there is a finite subfamily $\{U_{\alpha_i} : i = 1, 2, \dots, n\}$ which covers $\bigcup \{U_\alpha : \alpha \in \Lambda\}$. This the subfamily is also a cover of the set B .

Theorem (4.11): Every $\mathfrak{s}$ -g-coc-closed subset of $\mathfrak{s}$ -g-coc-compact space is $\mathfrak{s}$ -g-coc-compact relative.

Proof:

Let A be an $\mathfrak{s}$ -g-coc-closed subset of X . Let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of A by $\mathfrak{s}$ -g-coc-open subset of X . Now for each $x \in X - A$, there is a $\mathfrak{s}$ -g-coc-open set V_x such that $V_x \cap A$ is a finite. Since X is $\mathfrak{s}$ -g-coc-compact and the collection $\{U_\alpha : \alpha \in \Lambda\} \cup \{V_x : x \in X - A\}$ is a $\mathfrak{s}$ -g-coc-open cover of X , there exists a finite sub cover $\{U_{\alpha_i} : i = 1, \dots, n\} \cup \{V_{x_i} : i = 1, \dots, n\}$. Since $\bigcup_{i=1}^n (V_{x_i} \cap A)$ is finite, so for each $x_j \in (V_{x_i} \cap A)$, there is $U_{\alpha(x_j)} \in \{U_\alpha : \alpha \in \Lambda\}$ such that $x_j \in U_{\alpha(x_j)}$ and $j = 1, \dots, n$. Hence $\{U_{\alpha_i} : i = 1, \dots, n\} \cup \{U_{\alpha(x_j)} : j = 1, \dots, n\}$ is a finite sub cover of $\{U_\alpha : \alpha \in \Lambda\}$ and it covers A . Therefore, A is $\mathfrak{s}$ -g-coc-compact relative to X .

Definition(4.12):

- i. A space X is called CC if every $\mathfrak{s}$ -compact set in X is $\mathfrak{s}$ -closed.
- ii. A space X is called CC' if every $\mathfrak{s}$ -g-coc-compact set in X is $\mathfrak{s}$ -g-coc-closed.

Theorem(4.13):

For any space (X, μ) , then (X, μ^{gk}) is CC .

Proof.

Let $K \in C(X, \mu^{gk})$. As $\mu \subseteq \mu^{gk}$, then $C(X, \mu^{gk}) \subseteq C(X, \mu)$ and hence $K \in C(X, \mu)$. Thus, we have $X - K \in \mu^{gk}$, and hence K is $\mathfrak{s}$ -closed in the space (X, μ^{gk}) .

Theorem(4.14) :

Let X be a space. Then the following statements are equivalent:

- i. X is CC .
- ii. $\mu = \mu^{gk}$.

Proof. i $\Rightarrow$ ii

Since $\mu \subseteq \mu^{gk}$, it is sufficient to see that $\mu^{gk} \subseteq \mu$, let K is $\mathfrak{s}$ -compact then $X - K \in \mu^{gk}$, by i, $X - K \in \mu$, then $\mu^{gk} = \mu$

ii $\Rightarrow$ i Let $K \in C(X, \tau)$. Then $X - K \in \mu^{gk}$, and by ii, $X - K \in \tau$. Therefore, K is $\mathfrak{s}$ -closed in X .

Remark(4.15): It is clear that every CC space is CC' space but the converse is not true in general as the following example shows:

Example(4.16): Let $X = \{1, 2, 3\}$, $\tau = \{X, \phi, \{1\}, \{2\}, \{1, 2\}\}$, the $\mathfrak{s}$ -coc-open sets are discrete. $\{1\}$ is $\mathfrak{s}$ -compact set but not $\mathfrak{s}$ -closed then X is not CC space.

Definition (4.17):

Let $f: X \rightarrow Y$ be a function of a space X into a space Y then f is called $\mathfrak{s}$ -g-coc-compact function if $f^{-1}(A)$ is $\mathfrak{s}$ -compact set in X for every $\mathfrak{s}$ -coc-compact set A in Y .

Remark (4.18): Every $\mathfrak{s}$ -g-coc-compact function is $\mathfrak{s}$ -compact function.

Definition (4.19): Let $f: X \rightarrow Y$ be a function of a space X into a space Y then f is called $\mathfrak{s} - g - coc'$ -compact function if $f^{-1}(A)$ is $\mathfrak{s}$ -g-coc-compact set in X for every $\mathfrak{s}$ -g-coc-compact set A in Y .

Example (4.20): Every function from a finite space into any space is $\mathfrak{s} - g - coc'$ -compact function.

Remark (4.21): Every $\mathfrak{s} - g - coc'$ -compact function is $\mathfrak{s}$ -g-coc-compact function.

Remark (2.22) :

Let $f: X \rightarrow Y$ be a function for which X is CC then the following statements are equivalent:

- i. f is $\mathfrak{s}$ -continuous.
- ii. f is $\mathfrak{s}$ -g-coc-continuous.

Remark (2.23):

Let $f: X \rightarrow Y$ be a function for which X is CC then the following statements are equivalent:

- i. f is $\mathfrak{s}$ -continuous.
- ii. f is $\mathfrak{s}$ -g- coc' -continuous.

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