On Semigroup Ideals and Generalized Two Sided Reverse α -3- Derivation in Prime Near-Ring

<p>Authors Names Ikram A. Saed</p> <p>Article History Received on: 17/5/2020 Revised on: 18 /7/2020 Accepted on: 20/8/2020</p> <p>Keywords: near-ring , prime near-ring , semigroup ideal , α-3-derivation , two sided reverse α-3-derivation , generalized two sided reverse α-3-derivation .</p> <p>DOI:https://doi.org/10.29350/jops.2020.25. 4.1150</p>	<p>ABSTRACT</p> <p>Let N be a near –ring and α is a mapping on N .In this paper, we will introduce the notions of two sided reverse α-3-derivation and generalized two sided reverse α-3-derivation of N . Then we will study commutativity of N under some conditions determined on semigroup ideals of N .</p> <p>MSC: 16A70 , 16N60 , 16W25 .</p>
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1. Introduction

N. Argac [6] studied the commutativity of prime near-ring N using the notion of two sided α -derivation . N. L. Oukhtite and A. Raji [7] continued in the same line , they generalized some known results involving semigroup ideal and generalized two sided α -derivations . M. Ashraf et al. [8,9,10,11] studied the commutativity of near-ring N using the notions of n -derivations , (σ, τ) - n -derivations and generalized n -derivations . Hence, it should be interesting to study the commutativity of a near-ring N admitting some conditions on other n -additive mappings . E. F. Adhab [5] studied the commutativity of prime near-ring N using the notions of two sided α - n -derivation and generalized two sided α - n -derivation of near-ring N .

For more information see [12, 13, 14]

Let N be a near –ring and α is a mapping on N . This paper consists of two sections . In section one , we recall some known definitions and necessary lemmas that we will use it later in this

paper . In section two , we define the concepts of two sided reverse α -3-derivation and generalized two sided reverse α -3-derivation of N , also we determine some conditions of generalized two sided reverse α -3-derivation and semigroup ideals which make prime near-ring commutative ring .

2.Basic Concepts

Definition 2.1:[1] A right near-ring (resp. a left near-ring) is a nonempty set N equipped with two binary operations $+$ and $\cdot$ such that

- (i) $(N, +)$ is a group (not necessarily abelian)
- (ii) $(N, \cdot)$ is a semigroup .
- (iii) For all $x, y, z \in N$, we have

$$(x+y)z = xz + yz \text{ (resp. } z(x+y) = zx + zy \text{)}$$

Example 2.2:[1] Let G be a group (not necessarily abelian) then all mapping of G into itself form a right near-ring $M(G)$ with regard to point wise addition and multiplication by composite

Definition 2.3:[2] A near-ring N is called a prime near-ring if $aNb = \{0\}$, where $a, b \in N$, implies that either $a = 0$ or $b = 0$.

Definition 2.4:[2] Let N be a near-ring . The symbol Z will denote the multiplicative center of N , that is $Z = \{x \in N / xy = yx \text{ for all } y \in N\}$.

Definition 2.5:[2] Let R be a ring . Define a Lie product $[,]$ on R as follows

$$[x, y] = xy - yx , \text{ for all } x, y \in R .$$

Properties 2.6:[2] Let R be a ring , then for all $x, y, z \in R$, we have :

$$1-[x, yz] = y[x, z] + [x, y]z$$

$$2-[xy, z] = x[y, z] + [x, z]y$$

$$3-[x+y, z] = [x, z] + [y, z]$$

$$4-[x, y+z] = [x, y] + [x, z]$$

Definition 2.7:[2] A nonempty subset U of N will be called a semigroup right ideal (resp. semigroup left ideal) if $UN \subset U$ (resp. $NU \subset U$) and if U is both semigroup right ideal and semigroup left ideal , it be called a semigroup ideal .

Remark 2.8:[2] Let N be a near-ring

- (i) $N \times N \times \dots \times N$ forms a near-ring with regard to component wise addition and component wise multiplication .

(ii) If $U_1, U_2, \dots, U_n$ be nonzero semigroup right ideals (resp. semigroup left ideals) of N , then $U_1 \times U_2 \times \dots \times U_n$ forms a nonzero semigroup right ideals (resp. semigroup left ideals) of $N \times N \times \dots \times N$.

Definition 2.9:[3] Suppose that W is a near-ring . An 3-additive mapping $d : W \times W \times W \rightarrow W$ is called 3-derivation if the relations :

$$d(s_1 s_1', s_2, s_3) = d(s_1, s_2, s_3) s_1' + s_1 d(s_1', s_2, s_3)$$

$$d(s_1, s_2 s_2', s_3) = d(s_1, s_2, s_3) s_2' + s_2 d(s_1, s_2', s_3)$$

$$d(s_1, s_2, s_3 s_3') = d(s_1, s_2, s_3) s_3' + s_3 d(s_1, s_2, s_3')$$

hold for all $s_1, s_1', s_2, s_2', s_3, s_3' \in W$.

Example 2.10 : [4] Let S be a commutative near-ring .

Let us define

$$W = \left\{ \begin{pmatrix} r & u \\ 0 & 0 \end{pmatrix} : r, u, 0 \in S \right\}.$$

And $d : W \times W \times W \rightarrow W$

$$d\left(\begin{pmatrix} r_1 & u_1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} r_2 & u_2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} r_3 & u_3 \\ 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & r_1 r_2 r_3 \\ 0 & 0 \end{pmatrix}$$

Then d is 3-derivation of W .

Definition 2.11:[4] Suppose that W is a near-ring and d be 3- derivation of W . An 3-additive mapping $f : W \times W \times W \rightarrow W$ is said to be generalized 3-derivation of W associated with d if the relations

$$f(s_1 s_1', s_2, s_3) = f(s_1, s_2, s_3) s_1' + s_1 d(s_1', s_2, s_3)$$

$$f(s_1, s_2 s_2', s_3) = f(s_1, s_2, s_3) s_2' + s_2 d(s_1, s_2', s_3)$$

$$f(s_1, s_2, s_3 s_3') = f(s_1, s_2, s_3) s_3' + s_3 d(s_1, s_2, s_3')$$

hold for all $s_1, s_1', s_2, s_2', s_3, s_3' \in W$.

Example 2.12 : [4] Let S be a commutative near-ring .

Let us define

$$W = \left\{ \begin{pmatrix} 0 & r \\ 0 & u \end{pmatrix} : r, u, 0 \in S \right\}.$$

And $d, f : W \times W \times W \rightarrow W$,

$$d\left(\begin{pmatrix} 0 & r_1 \\ 0 & u_1 \end{pmatrix}, \begin{pmatrix} 0 & r_2 \\ 0 & u_2 \end{pmatrix}, \begin{pmatrix} 0 & r_3 \\ 0 & u_3 \end{pmatrix}\right) = \begin{pmatrix} 0 & r_1 r_2 r_3 \\ 0 & 0 \end{pmatrix}$$

$$f\left(\begin{pmatrix} 0 & r_1 \\ 0 & u_1 \end{pmatrix}, \begin{pmatrix} 0 & r_2 \\ 0 & u_2 \end{pmatrix}, \begin{pmatrix} 0 & r_3 \\ 0 & u_3 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ 0 & u_1 u_2 u_3 \end{pmatrix}$$

Then f is a generalized 3-derivation of W .

Lemma 2.13:[5] Let N be a prime near-ring and U be a nonzero semigroup right ideal (resp. semigroup left ideal) of N and x is an element of N such that $Ux = \{0\}$ (resp. $xU = \{0\}$) , then $x = 0$.

Lemma 2.14:[5] Let N be a prime near-ring and U be a nonzero semigroup ideal of N . If $x, y \in N$ and $xUy = \{0\}$, then $x = 0$ or $y = 0$.

Lemma 2.15:[5] Let N be a prime near-ring and Z contains a nonzero semigroup left ideal or nonzero semigroup right ideal , then N is a commutative ring .

3.Main Results

First we introduce the basic definitions in this paper

Definition 3.1: Let N be a near-ring. An 3-additive mapping $d : N \times N \times N \rightarrow N$ is said to be two sided reverse α -3-derivation if there exists a function $\alpha : N \rightarrow N$ such that the relations :

$$\begin{aligned} d(x_1'x_1, x_2, x_3) &= d(x_1, x_2, x_3)x_1' + \alpha(x_1)d(x_1', x_2, x_3) \\ &= d(x_1, x_2, x_3)\alpha(x_1') + x_1d(x_1', x_2, x_3) \end{aligned}$$

$$\begin{aligned} d(x_1, x_2'x_2, x_3) &= d(x_1, x_2, x_3)x_2' + \alpha(x_2)d(x_1, x_2', x_3) \\ &= d(x_1, x_2, x_3)\alpha(x_2') + x_2d(x_1, x_2', x_3) \end{aligned}$$

$$\begin{aligned} d(x_1, x_2, x_3'x_3) &= d(x_1, x_2, x_3)x_3' + \alpha(x_3)d(x_1, x_2, x_3') \\ &= d(x_1, x_2, x_3)\alpha(x_3') + x_3d(x_1, x_2, x_3') \end{aligned}$$

hold for all $x_1, x_1', x_2, x_2', x_3, x_3' \in N$.

Definition 3.2: Let N be a near-ring and $d : N \times N \times N \rightarrow N$ be a two sided reverse α -3-derivation of N . An 3-additive mapping $f : N \times N \times N \rightarrow N$ is said to be generalized two sided reverse α -3-derivation associated with two sided reverse α -3-derivation d if the relations :

$$\begin{aligned} f(x_1'x_1, x_2, x_3) &= d(x_1, x_2, x_3)x_1' + \alpha(x_1)f(x_1', x_2, x_3) \\ &= d(x_1, x_2, x_3)\alpha(x_1') + x_1f(x_1', x_2, x_3) \end{aligned}$$

$$\begin{aligned} f(x_1, x_2'x_2, x_3) &= d(x_1, x_2, x_3)x_2' + \alpha(x_2)f(x_1, x_2', x_3) \\ &= d(x_1, x_2, x_3)\alpha(x_2') + x_2f(x_1, x_2', x_3) \end{aligned}$$

$$\begin{aligned} f(x_1, x_2, x_3'x_3) &= d(x_1, x_2, x_3)x_3' + \alpha(x_3)f(x_1, x_2, x_3') \\ &= d(x_1, x_2, x_3)\alpha(x_3') + x_3f(x_1, x_2, x_3') \end{aligned}$$

hold for all $x_1, x_1', x_2, x_2', x_3, x_3' \in N$.

We begin with the following lemmas which are essential for developing the proofs of our main results, α will represent a homomorphism of N .

Lemma 3.3 : Let N be a near-ring and d be a two sided reverse α -3-derivation of N , then

$$\begin{aligned} d(x_1'x_1, x_2, x_3) &= x_1d(x_1', x_2, x_3) + d(x_1, x_2, x_3)\alpha(x_1') \\ &= \alpha(x_1)d(x_1', x_2, x_3) + d(x_1, x_2, x_3)x_1' \end{aligned}$$

$$\begin{aligned} d(x_1, x_2'x_2, x_3) &= x_2d(x_1, x_2', x_3) + d(x_1, x_2, x_3)\alpha(x_2') \\ &= \alpha(x_2)d(x_1, x_2', x_3) + d(x_1, x_2, x_3)x_2' \end{aligned}$$

$$\begin{aligned} d(x_1, x_2, x_3'x_3) &= x_3d(x_1, x_2, x_3') + d(x_1, x_2, x_3)\alpha(x_3') \\ &= \alpha(x_3)d(x_1, x_2, x_3') + d(x_1, x_2, x_3)x_3' \end{aligned}$$

hold for all $x_1, x_1', x_2, x_2', x_3, x_3' \in N$.

Lemma 3.4 : Let N be a near-ring and d be a two sided reverse α -3-derivation of N , then

$$\begin{aligned} (i) (\alpha(x_1)d(x_1', x_2, x_3) + d(x_1, x_2, x_3)x_1')y &= \alpha(x_1)d(x_1', x_2, x_3)y + d(x_1, x_2, x_3)x_1'y \\ (\alpha(x_2)d(x_1, x_2', x_3) + d(x_1, x_2, x_3)x_2')y &= \alpha(x_2)d(x_1, x_2', x_3)y + d(x_1, x_2, x_3)x_2'y \\ (\alpha(x_3)d(x_1, x_2, x_3') + d(x_1, x_2, x_3)x_3')y &= \alpha(x_3)d(x_1, x_2, x_3')y + d(x_1, x_2, x_3)x_3'y \end{aligned}$$

for all $x_1, x_1', x_2, x_2', x_3, x_3', y \in N$.

$$\begin{aligned} (ii) (x_1d(x_1', x_2, x_3) + d(x_1, x_2, x_3)\alpha(x_1'))y &= x_1d(x_1', x_2, x_3)y + d(x_1, x_2, x_3)\alpha(x_1')y \\ (x_2d(x_1, x_2', x_3) + d(x_1, x_2, x_3)\alpha(x_2'))y &= x_2d(x_1, x_2', x_3)y + d(x_1, x_2, x_3)\alpha(x_2')y \\ (x_3d(x_1, x_2, x_3') + d(x_1, x_2, x_3)\alpha(x_3'))y &= x_3d(x_1, x_2, x_3')y + d(x_1, x_2, x_3)\alpha(x_3')y \end{aligned}$$

for all $x_1, x_1', x_2, x_2', x_3, x_3', y \in N$.

Lemma 3.5 : Let N be a prime near-ring and d be a nonzero two sided reverse α -3-derivation of N . Let U_1, U_2, U_3 be a nonzero semigroup ideals of N . If $d(U_1, U_2, U_3) = \{0\}$, then $d(N, N, N) = \{0\}$.

Now, we will prove the main results :

Theorem 3.6 : Let N be a prime near-ring and d be a nonzero two sided reverse α -3-derivation of N . Let U_1, U_2, U_3 be a nonzero semigroup ideals of N . If $d(U_1, U_2, U_3) \subseteq Z$, then N is a commutative ring.

Proof : We are given that

$$d(u_1, u_2, u_3) \in Z \text{ for all } u_1 \in U_1, u_2 \in U_2, u_3 \in U_3. \quad (3.1)$$

Therefore

$$t d(u_1' u_1, u_2, u_3) = d(u_1' u_1, u_2, u_3) t \text{ for all } u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3, t \in N.$$

By Lemma 3.4 (ii) and defining property of d we get

$$\begin{aligned} & t u_1 d(u_1', u_2, u_3) + t d(u_1, u_2, u_3) \alpha(u_1') \\ &= u_1 d(u_1', u_2, u_3) t + d(u_1, u_2, u_3) \alpha(u_1') t \end{aligned}$$

for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3, t \in N$. Using (3.1) again, we obtain

$$d(u_1', u_2, u_3) t u_1 + d(u_1, u_2, u_3) t \alpha(u_1') = d(u_1', u_2, u_3) u_1 t + d(u_1, u_2, u_3) \alpha(u_1') t$$

for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3, t \in N$. (3.2)

Replacing t by $\alpha(u_1')$ in (3.2), we get

$$d(u_1', u_2, u_3) \alpha(u_1') u_1 = d(u_1', u_2, u_3) u_1 \alpha(u_1') \text{ for all } u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3.$$

i.e.; $d(u_1', u_2, u_3) N [\alpha(u_1'), u_1] = \{0\}$ for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3$.

Primeness of N yields that for each $u_1' \in U_1$, we get either $d(u_1', u_2, u_3) = 0$ for all $u_2 \in U_2, u_3 \in U_3$ or $[\alpha(u_1'), u_1] = 0$ for all $u_1 \in U_1$.

If $d(u_1', u_2, u_3) = 0$ for all $u_1' \in U_1, u_2 \in U_2, u_3 \in U_3$, then by Lemma 3.5 we conclude that $d(N, N, N) = \{0\}$, leading to a contradiction as d is a nonzero two sided reverse α -3-derivation of N . Therefore there exist $x_1 \in U_1, x_2 \in U_2, x_3 \in U_3$ all being nonzero such that $d(x_1, x_2, x_3) \neq 0$ and $\alpha(x_1) u = u \alpha(x_1)$ for all $u \in U_1$, replacing u by ut where $t \in N$, we get $U_1 [\alpha(x_1), t] = \{0\}$, for all $t \in N$. By Lemma 2.13 we get $\alpha(x_1) \in Z$.

Taking x_1 instead of u_1' , x_2 instead of u_2 , x_3 instead of u_3 in (3.2), we obtain

$$d(x_1, x_2, x_3) t u_1 = d(x_1, x_2, x_3) u_1 t \text{ for all } u_1 \in U_1, t \in N.$$

i.e.; $d(x_1, x_2, x_3) [t, u_1] = 0$, accordingly

$$d(x_1, x_2, x_3) N [t, u_1] = \{0\} \text{ for all } u_1 \in U_1, t \in N.$$

Primeness of N and $d(x_1, x_2, x_3) \neq 0$ yield that $U_1 \subseteq Z$, by Lemma 2.15 we conclude that N is a commutative ring.

Theorem 3.7 : Let N be a prime near-ring admitting a generalized two sided reverse α -3-derivation f associated with a nonzero two sided reverse α -3-derivation d . Let U_1, U_2, U_3 be a nonzero semigroup ideals of N . If $f([u_1, u_1'], u_2, u_3) = 0$, for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3$, then N is a commutative ring.

Proof : By our hypothesis we have

$$f([u_1, u_1'], u_2, u_3) = 0, \text{ for all } u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3.$$

Replacing u_1' by $u_1' u_1$ in preceding equation and using it again we get

$$d(u_1, u_2, u_3) [u_1, u_1'] = 0, \text{ for all } u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3.$$

i.e.; $d(u_1, u_2, u_3) u_1 u_1' = d(u_1, u_2, u_3) u_1' u_1$,

$$\text{for all } u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3 \quad (3.3)$$

Replacing u_1' by u_1'/r , where $r \in N$, in (3.3) and using it again we get

$d(u_1, u_2, u_3) u_1' [u_1, r] = 0$, for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3, r \in N$.

Therefore $d(u_1, u_2, u_3) U_1 [u_1, r] = \{0\}$, for all $u_1 \in U_1, u_2 \in U_2, u_3 \in U_3, r \in N$.

By Lemma 2.14, we conclude that for each $u_1 \in U_1$ either $u_1 \in Z$ or $d(u_1, u_2, u_3) = 0$ for all $u_2 \in U_2, u_3 \in U_3$. (3.4)

Let $x_1 \in U_1 \cap Z$, by Lemma 3.3 and defining property of d , we have for all $y \in N$.

$$\begin{aligned} d(yx_1, u_2, u_3) &= x_1 d(y, u_2, u_3) + d(x_1, u_2, u_3) \alpha(y) \\ &= d(x_1 y, u_2, u_3) \end{aligned}$$

$$= d(y, u_2, u_3) x_1 + \alpha(y) d(x_1, u_2, u_3)$$

for all $u_2 \in U_2, u_3 \in U_3, y \in N$. Which implies that

$$d(x_1, u_2, u_3) \alpha(y) = \alpha(y) d(x_1, u_2, u_3)$$

for all $u_2 \in U_2, u_3 \in U_3, y \in N$.

In view of equation (3.4) we get

$$\begin{aligned} d(u_1, u_2, u_3) \alpha(y) &= \alpha(y) d(u_1, u_2, u_3) \\ \text{for all } u_1 \in U_1, u_2 \in U_2, u_3 \in U_3, y \in N. \end{aligned} \quad (3.5)$$

On the other hand,

$$\begin{aligned} d(tx_1, u_2, u_3) &= d(x_1, u_2, u_3)t + \alpha(x_1)d(t, u_2, u_3) = d(x_1 t, u_2, u_3) \\ &= td(x_1, u_2, u_3) + d(t, u_2, u_3) \alpha(x_1) \end{aligned}$$

for all $u_2 \in U_2, u_3 \in U_3, t \in N$.

It follows that for all $u_2 \in U_2, u_3 \in U_3, t \in N$, we get

$$d(x_1, u_2, u_3)t + \alpha(x_1)d(t, u_2, u_3) = td(x_1, u_2, u_3) + d(t, u_2, u_3) \alpha(x_1) \quad (3.6)$$

In particular, taking $t \in U_1$ in (3.6) and using (3.5), we get

$$d(x_1, u_2, u_3)t = td(x_1, u_2, u_3) \text{ for all } t \in U_1, u_2 \in U_2, u_3 \in U_3.$$

Replacing t by ty , where $y \in N$, in the preceding equation and using it again to get

$$tyd(x_1, u_2, u_3) = d(x_1, u_2, u_3)ty = td(x_1, u_2, u_3)y$$

for all $t \in U_1, u_2 \in U_2, u_3 \in U_3, y \in N$, that is

$$t[d(x_1, u_2, u_3), y] = 0 \text{ for all } t \in U_1, u_2 \in U_2, u_3 \in U_3, y \in N.$$

Therefore

$U_1 [d(x_1, u_2, u_3), y] = \{0\}$, by Lemma 2.13 we get $d(x_1, u_2, u_3) \in Z$. According to (3.4) we conclude that $d(u_1, u_2, u_3) \in Z$ for all $u_1 \in U_1, u_2 \in U_2, u_3 \in U_3$, and hence N is a commutative ring by application of Theorem 3.6.

Corollary 3.8 : Let N be a prime near-ring admitting a nonzero two sided reverse α -3-derivation d . Let U_1, U_2, U_3 be a nonzero semigroup ideals of N . If $d([u_1, u_1'], u_2, u_3) = 0$, for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3$, then N is a commutative ring.

Theorem 3.9 : Let N be a prime near-ring admitting a nonzero generalized two sided reverse α -3-derivation f associated with a nonzero two sided reverse α -3-derivation d of N . Let U_1, U_2, U_3 be a nonzero semigroup ideals of N . If $f([u_1, u_1'], u_2, u_3) = \pm [u_1, u_1']$, for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3$, then N is a commutative ring.

Proof : By our hypothesis, we have

$$f([u_1, u_1'], u_2, u_3) = \pm [u_1, u_1'], \text{ for all } u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3. \quad (3.7)$$

Replacing u_1' by $u_1'u_1$ in (3.7) and using it again we get

$$d(u_1, u_2, u_3) \alpha([u_1, u_1']) = 0, \text{ for all } u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3.$$

i.e.; $d(u_1, u_2, u_3) \alpha(u_1) \alpha(u_1') = d(u_1, u_2, u_3) \alpha(u_1') \alpha(u_1)$, let $\alpha(U_1) = V_1$ since α is surjective, then V_1 is a semigroup ideal of N . Now let $\alpha(u_1') = v_1'$, where $v_1' \in V_1$ so we have for all $u_1 \in U_1, u_2 \in U_2, u_3 \in U_3, v_1' \in V_1$.

$$d(u_1, u_2, u_3) \alpha(u_1) v_1' = d(u_1, u_2, u_3) v_1' \alpha(u_1) \quad (3.8)$$

Replacing v_1' by v_1'/r , where $r \in N$, in (3.8) and using it again we get

$$d(u_1, u_2, u_3) v_1' [\alpha(u_1), r] = 0 \text{ for all } u_1 \in U_1, u_2 \in U_2, u_3 \in U_3, v_1' \in V_1, r \in N.$$

i.e.; $d(u_1, u_2, u_3) V_1 [\alpha(u_1), r] = \{0\}$ for all $u_1 \in U_1, u_2 \in U_2, u_3 \in U_3, r \in N$.

By Lemma 2.14, we get for all $u_1 \in U_1$, either $\alpha(u_1) \in Z$ or $d(u_1, u_2, u_3) = 0$

$$\text{for all } u_2 \in U_2, u_3 \in U_3 \quad (3.9)$$

Let $u \in U_1$ such that $d(u, u_2, u_3) = 0$, for all $u_2 \in U_2, u_3 \in U_3$, then

$$d(uu', u_2, u_3) = d(u', u_2, u_3) u + \alpha(u') d(u, u_2, u_3) = d(u', u_2, u_3) u \text{ and}$$

$$d(uu', u_2, u_3) = d(u', u_2, u_3) \alpha(u) + u' d(u, u_2, u_3) = d(u', u_2, u_3) \alpha(u)$$

for all $u' \in U_1, u_2 \in U_2, u_3 \in U_3$.

Combining both expressions of $d(uu', u_2, u_3)$, we obtain

$$d(u', u_2, u_3) (\alpha(u) - u) = 0 \text{ for all } u' \in U_1, u_2 \in U_2, u_3 \in U_3. \quad (3.10)$$

Replacing u' by wu' , where $w \in U_1$, in (3.10) and using it again to get

$$d(u', u_2, u_3) w (\alpha(u) - u) = 0 \text{ for all } u', w \in U_1, u_2 \in U_2, u_3 \in U_3.$$

i.e.; $d(u', u_2, u_3) U_1 (\alpha(u) - u) = \{0\}$ for all $u' \in U_1, u_2 \in U_2, u_3 \in U_3$.

By Lemma 2.14 we conclude that either $d(u', u_2, u_3) = 0$ for all $u' \in U_1, u_2 \in U_2, u_3 \in U_3$ or $\alpha(u) = u$.

If $d(u', u_2, u_3) = 0$ for all $u' \in U_1, u_2 \in U_2, u_3 \in U_3$, then by Lemma 3.5 we conclude $d = 0$, which contradicts our original assumption that $d \neq 0$.

Hence, we conclude that $\alpha(u) = u$. According to (3.9) we arrive at a conclusion.

For each $u_1 \in U_1$, either $\alpha(u_1) \in Z$ or $d(\alpha(u_1), u_2, u_3) = 0$ for all $u_2 \in U_2, u_3 \in U_3$

It follows for all $v_1 \in V_1$, we get either $v_1 \in Z$ or $d(v_1, u_2, u_3) = 0$ for all $u_2 \in U_2, u_3 \in U_3$.

Which is identical with the equation (3.4) in Theorem 3.7. Now arguing in the same way in corollary 3.8 we conclude that $d(v_1, u_2, u_3) \in Z$ for all $v_1 \in V_1, u_2 \in U_2, u_3 \in U_3$, and hence N is a commutative ring by application of Theorem 3.6

Conclusion

In present paper we introduce the notions of two sided reverse α -3-derivation and generalized two sided reverse α -3-derivation in near-ring and we see that a near-ring can be made commutative with help of generalized two sided reverse α -3-derivation and other conditions.

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