COMPUTATIONAL INVESTIGATION OF CONJUGATE HEAT TRANSFER IN CAVITY FILLED WITH SATURATED POROUS MEDIA

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\section*{ABSTRACT}

The conjugate natural convection heat transfer in a partially heated porous enclosure had been studied numerically. The governing dimensionless equations are solved using finite element method. Classical Darcy model have been used and the considering dimensionless parameters are modified Rayleigh number \(10 \leq Ra \leq 10^3\), finite wall thickness \(0.02 \leq D \leq 0.5\), thermal conductivity ratio \(0.1 \leq K_r \leq 10\), and the aspect ratio \(0.5 \leq A \leq 10\). The results are presented in terms of streamlines, isotherms and local and average Nusselt number. The results indicate that heat transfer can be enhanced by increasing the modified Rayleigh number, and thermal conductivity ratio. Wall thickness effects on the heat transfer mechanism had been studied and it is found that; as the Wall thickness increases, the conduction heat transfer mechanism will be dominated. Also, increasing aspect ratio will increase the stream function and reduced the heat transfer rate.

\textbf{Keywords:} partially heated, conjugate, porous cavity, aspect ratio, finite element method.

\section*{1. INTRODUCTION}

Due to the numerous engineering applications of natural convection heat transfer, a lot of investigation had been presented by many researchers. The industrial applications of natural convective fluid flow can be included in cooling storage, thermal solar-collectors, metals melting process, heat exchangers, chemical process and thermal insulation design (Basak, Singh, and Anandalakshmi 2014). Several studies regarding natural convection fluid flow in enclosure filled with pure fluid had been accomplished by (de Vahl Davis 1983; Mahapatra, Manna, and Ghosh 2015; Cianfrini et al. 2013; Kuhn and Oosthuizen 1987; Valencia and Frederick 1989; Ho and Chang 1994; Türkoglu and Yücel 1995; Alam et al. 2012). Mahapatra, Manna, and Ghosh (2015) identified numerically the optimum location of partially active wall for better heat transfer. A dimensionless correlation for heat transfer had been proposed for laminar natural convection in a rectangular enclosure filled by air (Cianfrini et al. 2013). Effect of aspect ratio had been examined numerically and experimentally in enclosure with four two-dimensional discrete heaters. A correlation for average number had been proposed in terms of modified Rayleigh number and aspect ratio had been reported by Ho and Chang (1994).

On the other hand, conjugate heat transfer in enclosures had been taken a lot of interest. Some of these studies presented by (Kaminski and Prakash 1986; Kuznetsov and Sheremet 2009; Ho and Yih 1987; Liaqat and Baytas 2001; Kahveci and Öztuna 1994; Türkoglu and Yücel 1996; Antar 2010; Cuckovic-Dzodzo, Dzodzo, and Pavlovic 1999; Antar and Baig 2009; Sambou et al. 2008). Kuznetsov and Sheremet (2009) studied numerically the heat transfer mechanism by convective-radiation with finite thickness wall heating at the bottom of the enclosure. The importance for the conjugate analysis of the thickness walls and how they may be give different results from non-conjugate analysis had been studied and examined by Liaqat and Baytas (2001). Natural convection heat transfer in porous enclosure had been studied by (Nithiarasu, Seetharamu, and Sundararajan 1997; Baytas and Pop 1999; Bin Kim 2001; Pakdee and Rattanadecho 2006; Basak et al. 2006; Pourshaghatghy, Hakaki-Fard, and Mahdavi-Nejad 2007; Sathiyamoorthy et al. 2007; Ramakrishna et al. 2013). A generalized model for double diffusive natural convection heat transfer in porous enclosure had been proposed by Nithiarasu, Seetharamu, and Sundararajan (1997). Natural convection phenomenon had been studied numerically in an inclined porous enclosure by Baytas and Pop (1999). Rectangular enclosure partially filled with a fluid-saturated porous medium with uniform heat generation had been examined numerically by Bin Kim (2001). Conjugate natural convection heat transfer in porous enclosure had been studied by (Baytas et al. 2001; Saeid 2008; Saeid 2007; Saleh and Hashim 2012; Saleh et al. 2011; Sheremet and Pop 2014; Ahmed et al. 2016). Baytaş et al. (2001) studied the natural convection in a square enclosure with two-finite thickness walls in the top and the bottom of the enclosure and they found that the heat transfer decreases with the increasing of the thermal conductivity ratio. Thermal non equilibrium model had been used by (Saeid 2008; Saeid 2007) and adopted to conjugate problem. Non-uniform heat generation in a square porous enclosure with finite thickness wall had been reported by (Saleh and Hashim 2012). Ahmed et al. (2016) demonstrated numerically by finite volume method the natural convection heat transfer in a square inclined enclosure with finite wall thickness on both sides. Biswas, Manna, and Mahapatra (2016) investigated numerically sinusoidal non-uniform heating effect over uniform heating from bottom of a square enclosure which is filled with fluid-saturated porous medium and cooled from sidewalls. Datta et al. (2016) showed how the adiabatic block body effect on the entropy generation and heat transfer enhancement. Recently, many
investigations focusing on the convection heat transfer in porous media using nanofluid as working fluid (Sheikholeslami and Seyednezhad 2018; Sheikholeslami and Rokni 2018; Sheikholeslami 2018; Sheikholeslami and Shehzad 2018; Al-Farhany and Abdulkadhim 2018).

It can be noticed from the literature review and according to the best author’s knowledge that there were limitations in the studies regarding partially active walls of enclosures so this is the motivation for the present work. In this way, the main objective of the present work is to describe the natural convection heat transfer in porous enclosure partially heated from the left side wall and how the finite thickness wall effect on heat transfer rate. The finite element method used to study the effect of various dimensionless parameters such as modified Rayleigh number, thermal conductivity ratio, thickness wall and the aspect ratio on the natural convection heat transfer characteristics. The results are presented in terms of streamline, isotherms, local and average Nusselt number.

2. MATHEMATICAL and COMPUTATIONAL MODEL

2.1 MATHEMATICAL FORMULATION

In this paper, two-dimension natural convection on porous cavity have been studied numerically with the effect of the partially heated conduction on vertical wall as shown in Fig. 1. The governing equations are subjected according to the assumptions:

1. The flow is considered to be two-dimensional fluid flow, laminar, and steady state.
2. The enclosure walls are impermeable.
3. The porous media is homogenous and isotropic.
4. The local thermal equilibrium is applied for porous matrix and the fluid.
5. Darcy model is applied for predictions of fluid flow inside the porous medium.
6. The internal heat generation assumed to be neglected.

The Continuity, momentum and energy of two-dimensional steady state natural convection in porous cavity equations are:

The Continuity equation is:
\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

(1)

X- and Y- momentum equations are:
\[
\frac{\partial U}{\partial Y} = -\frac{K}{\mu} \frac{\partial^2 P}{\partial X \partial Y} + Ra \frac{\partial T}{\partial X}
\]

(2)

The energy equation for porous cavity is:
\[
U \cdot \frac{\partial T}{\partial X} + V \cdot \frac{\partial T}{\partial Y} = \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}
\]

(3)

Energy equation at the wall:
\[
\frac{\partial^2 T_w}{\partial X^2} + \frac{\partial^2 T_w}{\partial Y^2} = 0
\]

(4)

The heat transfer at the walls are defined as in the following:
\[
Nu = \frac{1}{A} \left( \frac{\partial T}{\partial Y} \right)_w
\]

(5)

The non-dimensional parameters are:
\[
A = \frac{H}{L}, \quad X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad D = \frac{d}{L},
\]
\[
U = \frac{uL}{\alpha}, \quad V = \frac{vL}{\alpha}, \quad P = \frac{pL^2}{\rho \alpha^2}, \quad T = \frac{T - T_e}{T_h - T_e},
\]
\[
Ra = \frac{g \beta \Delta TKL}{\nu \alpha}, \quad k_f = \frac{k_f}{k_i}, \quad k_{eff} = ek_f + (1 - e)k_i
\]

Equations (1)-(6) are solved using non-dimensional initial boundary conditions:

\[
at X = D \quad U = V = 0 \quad \frac{\partial T_w}{\partial X} = k_f \frac{\partial T}{\partial X}
\]
\[
at X = 1 \quad U = V = 0 \quad T_e = 0
\]
\[
at X = 0 \quad T_h = 1
\]
\[
at Y = 0, A \quad U = V = 0 \quad \frac{\partial T}{\partial Y} = 0
\]

(8)

2.2 COMPUTATIONAL MODEL

In order to solve the problem with high accuracy and low computation time, different mesh had been tested for the minimum number of elements that leads to grid-independent solution. Fig. 2 illustrates the relation between the average Nusselt number and the resulting number of elements for square porous enclosure for all cases these tested at [Ra=1000, Da=10^{-3}, D=0.1, Kr=1]. It is shown that there is no effect on the average Nusselt number when the number of elements about (80000).

![Fig. 1 Schematic diagram of the present work](image)

![Fig. 2 Two dimensional computational domain within the triangle meshes type (left), the mesh independent study of average Nusselt number (right)](image)
The program had been validated regarding average Nusselt number with significant researcher as shown in Table 1. Moreover, for streamlines and isotherms contours, good agreements had been achieved with Saeid (2007) works as shown in Fig. 3.

Table 1: the validation of the present work with significant researchers

<table>
<thead>
<tr>
<th>Author</th>
<th>average Nusselt number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ra = 10</td>
</tr>
<tr>
<td>Moya, Ramos, and Sen (1987)</td>
<td>1.065</td>
</tr>
<tr>
<td>Beckermann, Viskanta, and Ramadhyani (1986)</td>
<td>no data</td>
</tr>
<tr>
<td>Al-Farhany and Turan (2011)</td>
<td>1.08</td>
</tr>
<tr>
<td>Ahmed et al. (2016)</td>
<td>1.093</td>
</tr>
<tr>
<td><strong>Present Work</strong></td>
<td><strong>1.08</strong></td>
</tr>
</tbody>
</table>

Fig. 3 Validation of the present work with Saeid work (Saeid 2007) in terms of isotherms and streamlines at D = 0.1 and 0.5 at Ra = 10³.

3. RESULTS AND DISCUSSIONS

3.1 MODIFIED RAYLEIGH NUMBER

Fig. 4 Demonstrates the isotherms (left) and streamlines (right) for various modified Rayleigh number and [D = 0.1, Kr = 1, and E = 0.5]. It can be noted that when modified Rayleigh number increase, the Nusselt number increase. This is recognized when modified Rayleigh number increase from Ra = 10 to Ra = 1000. \( \psi_{max} = 0.56 \) to \( \psi_{max} = 9.79 \). For example, Average Nusselt number increases from \( \overline{Nu_f} = 0.94847, \overline{Nu_w} = 1.6263 \) at Ra=100 to \( \overline{Nu_f} = 3.4729, \overline{Nu_w} = 6.2693 \) at Ra = 1000. The reason is due to increase flow circulation intensity when modified Rayleigh number increases. With respect to isotherms, it can be seen that when modified Rayleigh number Ra = 10, the isotherms have a uniform shape. This is due to weak effect of convective flow and in this case the conductive heat transfer is dominant. But, when modified Rayleigh number increases to Ra = 1000, the isotherms change their shapes obviously due to strong effect of convection heat transfer. Fig. 5 illustrates the profile of local Nusselt number for both the fluid (a) and the solid phase (b) for various values of modified Rayleigh number. As expected, the heat transfer rate enhanced significantly when Ra increases from 10 to Ra = 1000 due to the increasing of buoyancy and natural convection flow within the enclosure.

Fig. 4 Isotherm (left) and streamlines (right) for various modified Ra number, D = 0.1, Kr = 1, and E = 0.5

3.2 DIMENSIONLESS WALL THICKNESS EFFECT

Fig. 6 illustrates the isotherms (left) and streamlines (right) for various dimensionless wall thickness and [Ra = 1000, Kr = 1]. It may be noted that as the dimensionless wall thickness increases, the maximum stream function will decreases. For example, when the dimensionless wall thickness increases from D = 0.02 to D = 0.5, \( \psi_{max} = 15 \) into \( \psi_{max} = 6.4 \). Also, the Nusselt number will decrease because the conduction heat transfer is dominant with increasing wall thickness. For example, \( \overline{Nu_f} = 6.3921, \overline{Nu_w} = 12.973 \) at D=0.02 while \( \overline{Nu_f} = 1.2478, \overline{Nu_w} = 2.153 \) at D = 0.5. For the isotherms contours, it can be noticed that as the walls thickness increases, the isotherms pattern becomes more uniform which indicating that the effect of the conduction heat transfer mode becomes more significant. Moreover, the isotherms pattern shows that the heat is transferred from the left sidewall at the middle where the hot wall exists towards the cold right sidewall due to

...
the large temperature gradient and this result matches with the problem boundary conditions.

Fig. 7 displays effect of dimensionless wall thickness on the local Nusselt number for the fluid phase (a) and solid phase (b). It can be noted that conduction mode will be dominant as the wall thickness increasing which leads to reducing the rate of heat transfer on the enclosure.

Fig. 5 Profile of local Nusselt number for (a) the fluid phase and (b) along the active hot wall for various modified Rayleigh number

(a) $D = 0.02, \overline{Nu_f} = 6.3921, \overline{Nu_w} = 12.973$

(b) $D = 0.1, \overline{Nu_f} = 3.4729, \overline{Nu_w} = 6.2693$

Fig. 6 Isotherm (left) and streamlines (right) for various dimensionless wall thickness, Ra = 1000, Kr = 1, E = 0.5

(c) $D = 0.5, \overline{Nu_f} = 1.2478, \overline{Nu_w} = 2.153$

Fig. 7 Profile of local Nusselt number for the fluid phase and along the active hot wall for various finite wall thicknesses

3.3 THERMAL CONDUCTIVITY RATIO EFFECT

Fig. 8 displays the isotherms (left) and streamlines (right) for various thermal conductivity ratio and $[Ra = 1000, D = 0.1]$. It is known that the thermal conductivity ratio $K_r$ is defined as the ratio of the thermal conductivity of solid walls to the thermal conductivity of the fluid. Therefore, from this definition it can be obtained that when the thermal conductivity ratio $K_r$ is small, i.e., $K_r = 1$, the thermal conductivity of walls is small, too. So, the thermal resistance is high and as a result the average Nusselt number at solid walls is high, while the average Nusselt number of fluids is low. On the contrary, when the thermal conductivity ratio increases from $K_r = 1$ to $K_r = 10$, the thermal conductivity of solid walls increases, i.e., convection decreases, while it decreases for the fluid, i.e., convection increases. For example, when thermal conductivity ratio increases from $K_r = 1$ to $K_r = 10$, maximum stream function value will increases from $\psi_{max} = 9.7$ to $\psi_{max} = 15$ respectively. For this reason, the average Nusselt number at solid walls decreases from $\overline{Nu_w} = 6.3712$ at $K_r = 1$ to $\overline{Nu_w} = 1.7256$ at $K_r = 10$. On the other hand, average Nusselt number for the fluid phase increases
from \( \frac{Nu_f}{K_r} = 3.3715 \) at \( K_r = 1 \) to \( \frac{Nu_f}{K_r} = 9.9799 \) at \( K_r = 10 \). This result demonstrates that the heat transfer mechanism inside the enclosure is converted from conduction mode when \( K_r \) is small into convection mode when \( K_r \) is high. Fig. 9 illustrates the effect of thermal conductivity ratio for the fluid phase (a) and along the solid wall (b). The results indicate that when the thermal conductivity ratio increases, the thermal conductivity of the fluid phase decreases. This leads to enhancing of the natural convection heat transfer and increasing the local Nusselt number for the fluid phase. However, this behavior is completely reversed for the Nusselt number along the solid wall. Actually, the local Nusselt number along the hot wall decreases as the thermal conductivity ratio increases, which leads to enhancing in the conduction effect.

![Isotherm and streamlines](image)

(a) \( K_r = 0.1, \frac{Nu_f}{K_r} = 2.8531, \frac{Nu_w}{K_r} = 9.4808 \)

(b) \( K_r = 1, \frac{Nu_f}{K_r} = 3.3715, \frac{Nu_w}{K_r} = 6.3712 \)

(c) \( K_r = 10, \frac{Nu_f}{K_r} = 4.9799, \frac{Nu_w}{K_r} = 1.7256 \)

Fig. 8 Isotherm (left) and streamlines (right) for various thermal conductivity ratio, \( Ra = 1000, D = 0.1, E = 0.5 \)

![Fig. 10](image)

Fig. 10 Streamlines (Top) and isotherms (bottom) for various aspect ratio at \( D = 0.1, Ra = 10^3 \).
The present work illustrates numerically the natural convection heat transfer in enclosure filled with porous media using Darcy model. The results can be summarized as follow:

1. When the dimensionless wall thickness increases, the convection mechanism will be converted into conduction mode. This will reduce the Nusselt number leading to reduce the rate of heat transfer.
2. When the thermal conductivity ratio increases, the local Nusselt number for the fluid phase will increase. While a reverse behavior for local Nusselt number along the heated wall.
3. As the Rayleigh number increases, the heat transfer rate will be enhanced, as a results, local Nusselt number for both fluid and solid phase will be increased.
4. Increasing of the aspect ratio makes the flow strength increases and the heat transfer decreases.

3. CONCLUSIONS

As the Rayleigh number increases, the heat transfer rate will increase and the heat transfer decreases. Increasing of the aspect ratio makes the flow strength increases and the heat transfer decreases.

4. CONCLUSIONS

The present work illustrates numerically the natural convection heat transfer in enclosure filled with porous media using Darcy model. The results can be summarized as follow:

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4. Increasing of the aspect ratio makes the flow strength increases and the heat transfer decreases.


defined symbols

- $\alpha$: effective thermal diffusivity, $\text{m}^2\text{s}^{-1}$
- $\beta$: coefficient of thermal expansion, $\text{K}^{-1}$
- $\nu$: kinematic viscosity, $\text{m}^2\text{s}^{-1}$
- $\rho$: density, $\text{kg m}^{-3}$

Subscripts

- $c$: cold
- $eff$: effective
- $f$: fluid
- $h$: hot
- $w$: wall

REFERENCES


