Symmetric Reverse Gamma *-4-Centralizers on Semiprime Gamma Rings with Involution

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ABSTRACT

In this paper, the symmetric left(right) reverse Γ^* -4-centralizer of a Γ -ring M with involution is presented and studied. Then we proved that the 4-additive mapping $T : M \times M \times M \times M \rightarrow M$ is a reverse Γ^* -4-centralizer of M if it satisfies one of these conditions :

$$(i) \quad T((r \circ y)_\gamma, r_2, r_3, r_4) = (T(r, r_2, r_3, r_4) \circ y^*)_\gamma \\ = (r^* \circ T(y, r_2, r_3, r_4))_\gamma$$

$$(ii) \quad T(r^3, r_2, r_3, r_4) = r^* \gamma T(r, r_2, r_3, r_4) \beta r^*$$

$$(iii) \quad T(r, r_2, r_3, r_4) \gamma y^* = r^* \gamma T(y, r_2, r_3, r_4)$$

for all $y, r, r_2, r_3, r_4 \in M$ and $\gamma, \beta \in \Gamma$.

1. Introduction

Majeed and Al-Taay in 2010 [1] proved many results of symmetric reverse $*$ -centralizer of $*$ -ring. Saed in 2016 [2] introduced the notion of double reverse θ^* -centralizer of rings with involution and proved basic properties of this type of mapping. Again Saed in 2016 [3] introduced the notion of Jordan $(\theta, \theta)^*$ -derivation pairs of rings with involution and proved basic properties of this type of mapping. Faraj and Super in 2020 [4] these results are studied by using the concept of symmetric reverse $*$ -n-centralizer.

Let M be a Γ -ring with involution. This paper is organized as follows. In section two, we recall some well-known definitions, examples and results that will be used in this paper. In section three, we present the notion of symmetric left(right) reverse Γ^* -4-centralizer of M , and we worked on some results, which give the two cases that (left and right) reverse Γ^* -4-centralizer have come up with a

concept symmetric reverse Γ^* -4-centralizer and showed some related results of the present concept under certain conditions .

2. Basic Concept

Definition 2.1:[5]

Assume M and Γ be additive abelian groups. If there exists a mapping $M \times \Gamma \times M \rightarrow M : (a, \alpha, b) \rightarrow a \alpha b$ which satisfies the conditions :

for every $a, b, c \in M, \alpha, \beta \in \Gamma$:

$$(i) \quad \begin{aligned} (a+b) \alpha c &= a \alpha c + b \alpha c \\ a(\alpha+\beta)b &= a \alpha b + a \beta b \\ a \alpha (b+c) &= a \alpha b + a \alpha c \end{aligned}$$

$$(ii) \quad (a \alpha b) \beta c = a \alpha (b \beta c)$$

Where M is refer to as a Γ -ring .

Example 2.2:

$$\text{Let } M = \left\{ \begin{pmatrix} a & b & 0 \\ 0 & c & 0 \\ 0 & 0 & 0 \end{pmatrix} : a, b, c \in Z \right\}, \text{ and } \Gamma = \left\{ \begin{pmatrix} n & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : n \in Z \right\}$$

We use the usual addition and multiplication on matrices of

$M \times \Gamma \times M \rightarrow M$, then M is a Γ -ring .

Definition 2.3: [6]

Let M , be a Γ -ring . The set $Z(M) = \{ x \in M : x \gamma y = y \gamma x, \text{ for all } y \in M \text{ and } \gamma \in \Gamma \}$ is called the center of the Γ - ring M .

Definition 2.4: [6]

A Γ -ring M is called semiprime if $a \Gamma M \Gamma a = \{0\}$ implies that $a=0$, for $a \in M$.

Definition 2.5: [6]

A Γ -ring M is called 2-torsion-free if $2a=0$ implies $a=0$, for $a \in M$.

Definition 2.6: [6]

A Γ -ring M is called a commutative if $a \gamma b = b \gamma a$, for all $a, b \in M$ and $\gamma \in \Gamma$.

Definition 2.7: [7]

Let M be a Γ -ring , for any $x, y \in M$ and $\alpha \in \Gamma$, the symbol

$[x, y]_\alpha = x \alpha y - y \alpha x$, will denote the commutator .

$(x \circ y)_\alpha = x \alpha y + y \alpha x$, will denote the additive group commutator .

Lemma 2.8: [7]

If M is a Γ -ring, for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$ then :

- (i) $[a, b]_{\alpha} + [b, a]_{\alpha} = 0$
- (ii) $[a + b, c]_{\alpha} = [a, c]_{\alpha} + [b, c]_{\alpha}$
- (iii) $[a, b + c]_{\alpha} = [a, b]_{\alpha} + [a, c]_{\alpha}$
- (iv) $[a, b]_{\alpha + \beta} = [a, b]_{\alpha} + [a, b]_{\beta}$
- (v) $[a \beta b, c]_{\alpha} = a \beta [b, c]_{\alpha} + [a, c]_{\alpha} \beta b + a \beta c \alpha b - a \alpha c \beta b$.

Definition 2.9: [7]

An additive mapping $(x\alpha x) \rightarrow (x\alpha x)^*$ on a Γ -ring M is called an involution if $(x\alpha y)^* = y^* \alpha x^*$ and $(x\alpha x)^{**} = x\alpha x$ for all $x, y \in M$ and $\alpha \in \Gamma$. A Γ -ring M equipped with an involution is called a Γ -ring M with involution (also known as Γ^* -ring).

Definition 2.10: [7]

An additive mapping $T: M \rightarrow M$ is left (right) reverse Γ^* -centralizer of a Γ -ring M with involution if $T(y\alpha x) = T(x) \alpha y^*$ ($T(y\alpha x) = x^* \alpha T(y)$) for all $x, y \in M$ and $\alpha \in \Gamma$.

Definition 2.11: [7]

A reverse Γ^* -centralizer of a Γ -ring M with involution is an additive mapping which is both a left and right reverse Γ^* -centralizer.

Example 2.12:

Let F be a field and $D_3(F)$ be a set of all diagonal matrices of order 3 with respect to the usual operation of addition and multiplication, then $D_3(F)$ is a commutative ring.

$T: D_3(F) \rightarrow D_3(F)$ be an additive mapping defined as

$$T(x) = T\left(\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad x, y \in F$$

$$\text{Define } y = \begin{bmatrix} x_1 & 0 & 0 \\ 0 & y_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad y^* = \begin{bmatrix} x_1 & 0 & 0 \\ 0 & y_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \alpha = \left\{ \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} : \alpha_1 \in \Gamma \right\}, \text{ then}$$

$T(y\alpha x) = T(x) \alpha y^*$. Hence T is a reverse Γ^* -centralizer.

3. Symmetric Reverse Γ^* -4-centralizers

First, we introduce the basic definitions in this paper

Definition 3.1:

Let M , be the Γ -ring with involution. An 4-additive mapping $T: M \times M \times M \times M \rightarrow M$ is said to be left reverse Γ^* -4-centralizer if the following equations hold for all $y, r_1, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$.

$$T_1(r_1 \gamma y, r_2, r_3, r_4) = T_1(y, r_2, r_3, r_4) \gamma r_1^*$$

$$T_2(r_1, r_2 \gamma y, r_3, r_4) = T_2(r_1, y, r_3, r_4) \gamma r_2^*$$

$$T_3(r_1, r_2, r_3 \gamma y, r_4) = T_3(r_1, r_2, y, r_4) \gamma r_3^*$$

$$T_4(r_1, r_2, r_3, r_4 \gamma y) = T_4(r_1, r_2, r_3, y) \gamma r_4^*$$

T is said to be a symmetric left reverse Γ^* -4-centralizer if all the above equations are equivalent to each other . That is ,

$$T(r_1 \gamma y, r_2, r_3, r_4) = T(y, r_2, r_3, r_4) \gamma r_1^*$$

for all $y, r_1, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$.

Definition 3.2:

Let M , be the Γ -ring with involution . An 4-additive mapping $T : M \times M \times M \times M \rightarrow M$ is said to be right reverse Γ^* -4-centralizer if the following equations hold for all $y, r_1, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$.

$$T_1(r_1 \gamma y, r_2, r_3, r_4) = y^* \gamma T_1(r_1, r_2, r_3, r_4)$$

$$T_2(r_1, r_2 \gamma y, r_3, r_4) = y^* \gamma T_2(r_1, r_2, r_3, r_4)$$

$$T_3(r_1, r_2, r_3 \gamma y, r_4) = y^* \gamma T_3(r_1, r_2, r_3, r_4)$$

$$T_4(r_1, r_2, r_3, r_4 \gamma y) = y^* \gamma T_4(r_1, r_2, r_3, r_4)$$

T is said to be a symmetric right reverse Γ^* -4-centralizer if all the above equations are equivalent to each other . That is ,

$$T(r_1 \gamma y, r_2, r_3, r_4) = y^* \gamma T(r_1, r_2, r_3, r_4)$$

for all $y, r_1, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$.

So , M is called symmetric reverse Γ^* -4-centralizer if M is symmetric left reverse Γ^* -4-centralizer and right reverse Γ^* -4-centralizer together .

Example 3.3:

Consider $M = \left\{ \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} : a, b, c \in \mathbb{C} \right\}$, where $\mathbb{C}$ is a ring of complex numbers . Clearly , M is a non-commutative ring under the usual addition and multiplication of matrices .

A map $T : M \times M \times M \times M \rightarrow M$ is defined by :

$$T \left\{ \begin{pmatrix} 0 & a_1 & b_1 \\ 0 & 0 & c_1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_2 & b_2 \\ 0 & 0 & c_2 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_3 & b_3 \\ 0 & 0 & c_3 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_4 & b_4 \\ 0 & 0 & c_4 \\ 0 & 0 & 0 \end{pmatrix} \right\} =$$

$$\begin{pmatrix} 0 & 0 & c_1 c_2 c_3 c_4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \text{ for all } \begin{pmatrix} 0 & a_1 & b_1 \\ 0 & 0 & c_1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_2 & b_2 \\ 0 & 0 & c_2 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_3 & b_3 \\ 0 & 0 & c_3 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_4 & b_4 \\ 0 & 0 & c_4 \\ 0 & 0 & 0 \end{pmatrix} \in M$$

Such that
$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}^* = \begin{pmatrix} 0 & c & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix}$$

And
$$\Gamma = \left\{ \begin{pmatrix} 0 & n & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : n \in Z \right\}$$

Then , T is a symmetric left reverse Γ^* -4-centralizer and also it is a right reverse Γ^* -4-centralizer

Lemma 3.4:

Let M be a semiprime Γ -ring with involution , let $a \in M$ be a fixed element , and let $T(r, r_2, r_3, r_4) = a\gamma r^* + r^* \gamma a$ satisfy

$T((r \circ y)_\beta, r_2, r_3, r_4) = (T(r, r_2, r_3, r_4) \circ y^*)_\beta = (r^* \circ T(y, r_2, r_3, r_4))_\beta$ for all $y, r, r_2, r_3, r_4 \in M$ and $\gamma, \beta \in \Gamma$ then $a \in Z(M)$.

Proof :

$$T((r \circ y)_\beta, r_2, r_3, r_4) = (T(r, r_2, r_3, r_4) \circ y^*)_\beta = (r^* \circ T(y, r_2, r_3, r_4))_\beta \quad (3.1)$$

By hypothesis $T(r, r_2, r_3, r_4) = a\gamma r^* + r^* \gamma a$

$$T((r \circ y)_\beta, r_2, r_3, r_4) = (a\gamma r^* + r^* \gamma a)_\beta y^* + y^* \beta (a\gamma r^* + r^* \gamma a) \quad (3.2)$$

Let $r = r\beta y + y\beta r$ in hypothesis , we get

$$T(r, r_2, r_3, r_4) = a\gamma (r\beta y + y\beta r)^* + (r\beta y + y\beta r)^* \gamma a \quad (3.3)$$

Then , by (3.1) and (3.2) , we have

$$a\gamma y^* \beta r^* + r^* \beta y^* \gamma a - r^* \gamma a \beta y^* - y^* \beta a \gamma r^* = 0$$

$$(a\gamma y^* - y^* \gamma a) \beta r^* + r^* \beta (y^* \gamma a - a\gamma y^*) = 0$$

This implies that $[[a, y^*]_\gamma, r^*]_\beta = 0$

By [Lemma 2.3 , 4] , we get $a \in Z(M)$.

Lemma 3.5:

Let M be a semiprime Γ -ring with involution . If every mappings T of M satisfy

$$T((r \circ y)_\gamma, r_2, r_3, r_4) = (T(r, r_2, r_3, r_4) \circ y^*)_\gamma = (r^* \circ T(y, r_2, r_3, r_4))_\gamma$$

for all $y, r, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$. Then T maps $Z(M)$ into $Z(M)$.

Proof :

Let $a = T(c, r_2, r_3, r_4)$ for $c \in Z(M)$ and $r, r_2, r_3, r_4 \in M$.

$$2T(c\beta r, r_2, r_3, r_4) = T(c\beta r + r\beta c, r_2, r_3, r_4) = T(c, r_2, r_3, r_4) \beta r^* +$$

$$\begin{aligned}
r^* \beta T(c, r_2, r_3, r_4) &= a \beta r^* + r^* \beta a \\
T(r\gamma y + y\gamma r, r_2, r_3, r_4) &= 2 T(c\beta(r\gamma y + y\gamma r), r_2, r_3, r_4) \\
&= 2[T(c\beta r, r_2, r_3, r_4) \gamma y^* + y^* \gamma T(c\beta r, r_2, r_3, r_4)] \\
&= 2[T(c\gamma y, r_2, r_3, r_4) \beta r^* + r^* \beta T(c\gamma y, r_2, r_3, r_4)] \\
&= T(c\beta r, r_2, r_3, r_4) \gamma y^* + y^* \gamma T(c\beta r, r_2, r_3, r_4) \\
&= T(c\gamma y, r_2, r_3, r_4) \beta r^* + r^* \beta T(c\gamma y, r_2, r_3, r_4) \\
&= T(r, r_2, r_3, r_4) \gamma y^* + y^* \gamma T(r, r_2, r_3, r_4) \\
&= T(y, r_2, r_3, r_4) \gamma r^* + r^* \gamma T(y, r_2, r_3, r_4)
\end{aligned}$$

For all $y, r, r_2, r_3, r_4 \in M$ and $\gamma, \beta \in \Gamma$.

By Lemma 3.4, we get : $a \in Z(M)$.

Theorem 3.6:

Let M be a 2-torsion free semiprime Γ -ring with involution, and $T : M \times M \times M \times M \rightarrow M$ be an 4-additive mapping which satisfies :

$T((r \circ y)_\gamma, r_2, r_3, r_4) = (T(r, r_2, r_3, r_4) \circ y^*)_\gamma = (r^* \circ T(y, r_2, r_3, r_4))_\gamma$ for all $y, r, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$. Then, T is a reverse Γ^* -4-centralizer of M .

Proof :

$$\begin{aligned}
T((r \circ y)_\gamma, r_2, r_3, r_4) &= (T(r, r_2, r_3, r_4) \circ y^*)_\gamma = (r^* \circ T(y, r_2, r_3, r_4))_\gamma \\
&= T(r\gamma y + y\gamma r, r_2, r_3, r_4) = \\
T(r, r_2, r_3, r_4) \gamma y^* + y^* \gamma T(r, r_2, r_3, r_4) &= T(y, r_2, r_3, r_4) \gamma r^* + r^* \gamma T(y, r_2, r_3, r_4)
\end{aligned}$$

Replacing $y = (r \circ y)_\beta$ in the last relation where $\beta \in \Gamma$, we have :

$$\begin{aligned}
T(r, r_2, r_3, r_4) \gamma (r \beta y + y \beta r)^* + (r \beta y + y \beta r)^* \gamma T(r, r_2, r_3, r_4) &= \\
T(r, r_2, r_3, r_4) \gamma y^* \beta r^* + y^* \gamma T(r, r_2, r_3, r_4) \beta r^* + r^* \beta T(r, r_2, r_3, r_4) \gamma y^* + r^* \beta y^* \gamma T(r, r_2, r_3, r_4) &=
\end{aligned}$$

This implies that :

$$\begin{aligned}
T(r, r_2, r_3, r_4) \gamma r^* \beta y^* + y^* \beta r^* \gamma T(r, r_2, r_3, r_4) &= \\
y^* \gamma T(r, r_2, r_3, r_4) \beta r^* + r^* \beta T(r, r_2, r_3, r_4) \gamma y^* &. \text{ Then :}
\end{aligned}$$

$$(T(r, r_2, r_3, r_4) \circ ((r \circ y)_\beta)^*)_\gamma = (T(r, r_2, r_3, r_4) \circ y^*)_\gamma \beta r^*$$

Also, we have :

$$[T(r, r_2, r_3, r_4), r^*]_\beta \gamma y^* = y^* \gamma [T(r, r_2, r_3, r_4), r^*]_\beta, \text{ we get :}$$

$$[T(r, r_2, r_3, r_4), r^*]_\beta \in Z(M).$$

Now, one will show that $[T(r, r_2, r_3, r_4), r^*]_\beta = 0$, and let $c \in Z(M)$

$$\begin{aligned} 2T(c\delta r, r_2, r_3, r_4) &= T(c\delta r + r\delta c, r_2, r_3, r_4) \\ &= T(c, r_2, r_3, r_4)\delta r^* + r^*\delta T(c, r_2, r_3, r_4) = 2T(r, r_2, r_3, r_4)\delta c^* \end{aligned}$$

By Lemma 3.5, we have

$$T(c\delta r, r_2, r_3, r_4) = T(r, r_2, r_3, r_4)\delta c^* = T(c, r_2, r_3, r_4)\delta r^*$$

Also, for all $c \in Z(M)$, one takes that

$$\begin{aligned} [T(r, r_2, r_3, r_4), r^*]_\beta \delta c^* &= T(r, r_2, r_3, r_4) \beta r^* \delta c^* - r^* \beta T(r, r_2, r_3, r_4) \delta c^* = T(r, r_2, r_3, r_4) \delta c^* \\ &\quad c^* \beta r^* - r^* \beta T(r, r_2, r_3, r_4) \delta c^* = T(c, r_2, r_3, r_4) \beta r^{*2} - r^* \beta T(c, r_2, r_3, r_4) \delta r^* \\ &= T(c, r_2, r_3, r_4) \beta r^* \delta r^* - r^* \beta T(c, r_2, r_3, r_4) \delta r^* \\ &= [T(c, r_2, r_3, r_4), r^*]_\beta \delta r^* \end{aligned}$$

For all $c \in Z(M)$, also one gets $T(c, r_2, r_3, r_4) \in Z(M)$, then

$$\begin{aligned} &= T(c, r_2, r_3, r_4) \beta r^* \delta r^* - T(c, r_2, r_3, r_4) \beta r^* \delta r^* \\ &= T(c, r_2, r_3, r_4) \beta r^{*2} - T(c, r_2, r_3, r_4) \beta r^{*2} \end{aligned}$$

One other hand, one will show that,

$$\begin{aligned} 2T(r^2, r_2, r_3, r_4) &= T(r\gamma r + r\gamma r, r_2, r_3, r_4) \\ &= T(r, r_2, r_3, r_4)\gamma r^* + r^*\gamma T(r, r_2, r_3, r_4) \\ &= 2r^*\gamma T(r, r_2, r_3, r_4) = 2T(r, r_2, r_3, r_4)\gamma r^* \end{aligned}$$

Theorem 3.7:

Assume that M be a 2-torsion free semiprime Γ -ring with involution with an identity element, and $T : M \times M \times M \times M \rightarrow M$ be an 4-additive mapping such that : $T(r^3, r_2, r_3, r_4) = r^*\gamma T(r, r_2, r_3, r_4)\beta r^*$. Then T is a reverse Γ^* -4-centralizer of M . for all $r, r_2, r_3, r_4 \in M$ and $\gamma, \beta \in \Gamma$

Proof :

$$\text{Since } T(r^3, r_2, r_3, r_4) = r^*\gamma T(r, r_2, r_3, r_4)\beta r^* \quad (3.4)$$

Multiply involution both sides to (3.4) to get the following

$$(T(r^3, r_2, r_3, r_4))^* = r\gamma (T(r, r_2, r_3, r_4))^*\beta r$$

for all $r, r_2, r_3, r_4 \in M$ and $\gamma, \beta \in \Gamma$.

Suppose that $F : M \times M \times M \times M \rightarrow M$, then

$F(r, r_2, r_3, r_4) = (T(r, r_2, r_3, r_4))^*$, and also we get

$$\begin{aligned} F(r^3, r_2, r_3, r_4) &= (T(r^3, r_2, r_3, r_4))^* = (r^* \gamma T(r, r_2, r_3, r_4) \beta r^*)^* \\ &= r \gamma (T(r, r_2, r_3, r_4))^* \beta r = r \gamma F(r, r_2, r_3, r_4) \beta r \end{aligned}$$

We have F is Γ -4-centralizer

$F(r\gamma y, r_2, r_3, r_4) = r \gamma F(y, r_2, r_3, r_4) = F(r, r_2, r_3, r_4) \gamma y$. Then ,

$$\begin{aligned} (T(r\gamma y, r_2, r_3, r_4))^* &= F(r\gamma y, r_2, r_3, r_4) = r \gamma F(y, r_2, r_3, r_4) \\ &= r \gamma (T(y, r_2, r_3, r_4))^* , \text{ for all } y, r, r_2, r_3, r_4 \in M \text{ and } \gamma \in \Gamma . \quad (3.5) \end{aligned}$$

Also ,

$$\begin{aligned} (T(r\gamma y, r_2, r_3, r_4))^* &= F(r\gamma y, r_2, r_3, r_4) = F(r, r_2, r_3, r_4) \gamma y \\ &= (T(r\gamma y, r_2, r_3, r_4))^* \gamma y , \text{ for all } y, r, r_2, r_3, r_4 \in M \text{ and } \gamma \in \Gamma . \quad (3.6) \end{aligned}$$

Multiply involution both sides to (3.5) and (3.6) to get

$$T(r\gamma y, r_2, r_3, r_4) = T(y, r_2, r_3, r_4) \gamma r^* = y^* \gamma T(r, r_2, r_3, r_4)$$

Theorem 3.8:

Suppose that M is a semiprime Γ -ring with involution , and $T : M \times M \times M \times M \rightarrow M$ is an 4-additive mapping . If $T(r, r_2, r_3, r_4) \gamma y^* = r^* \gamma T(y, r_2, r_3, r_4)$ for all $y, r, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$, then M is a left reverse Γ^* -4-centralizer of M .

Proof :

$$T(r, r_2, r_3, r_4) \gamma y^* = r^* \gamma T(y, r_2, r_3, r_4) \quad (3.7)$$

Calculating the following equation and by (3.7) , we have

$$\begin{aligned} T(r + y, r_2, r_3, r_4) \gamma z^* - T(r, r_2, r_3, r_4) \gamma z^* - T(y, r_2, r_3, r_4) \gamma z^* \\ &= (r + y)^* \gamma T(z, r_2, r_3, r_4) - r^* \gamma T(z, r_2, r_3, r_4) - y^* \gamma T(z, r_2, r_3, r_4) \\ &= ((r + y)^* - r^* - y^*) \gamma T(z, r_2, r_3, r_4) \\ &= (r^* + y^* - r^* - y^*) \gamma T(z, r_2, r_3, r_4) \end{aligned}$$

$$\text{This implies that } [T(r + y, r_2, r_3, r_4) - T(r, r_2, r_3, r_4) - T(y, r_2, r_3, r_4)] \gamma z^* = 0 \quad (3.8)$$

Now , let $z = z^*$ in (3.8) to get

$$[T(r + y, r_2, r_3, r_4) - T(r, r_2, r_3, r_4) - T(y, r_2, r_3, r_4)] \gamma z = 0$$

$$\text{for all } y, r, r_2, r_3, r_4 \in M \text{ and } \gamma \in \Gamma \quad (3.9)$$

Since M is semiprime ring , one obtains that

$$T(r + y, r_2, r_3, r_4) = T(r, r_2, r_3, r_4) + T(y, r_2, r_3, r_4)$$

Similarly , one calculates the relation

$$(T(y\gamma r, r_2, r_3, r_4) - T(r, r_2, r_3, r_4) \gamma y^*) \beta z^*, \text{ then } T \text{ is a left reverse } \Gamma^*\text{-4-centralizer of } M.$$

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