Accounts for the groups $\mathcal{SL}(2, \mathfrak{U})$, $\mathfrak{U} = 41$ and 43

Authors Names	ABSTRACT
<p>a. Sherouk Awad Khalaf b. Niran Sabah Jasim</p> <p>Article History Received on: 13 / 6/2021 Revised on: 1/ 6/2021 Accepted on: 2/7/2021</p> <p>Keywords: Circular retail, special linear group, ordinary character table, character table of rational representations.</p> <p>DOI: https://doi.org/10.29350/jops.2021.26.4.1353</p>	<p>The circular retail for the groups $\mathcal{SL}(2, \mathfrak{U})$ where $\mathfrak{U} = 41$ and 43 was compute in this paper from the ordinary character table and the character table (ch.t.) of rational representations (r.rep.) for each group.</p>

1. Introduction

The $\mathcal{SL}(n, F)$ is the subgroup of $GL(n, F)$ which contains all matrices of determinant one over the field F , [7,9]. By using the intellect which the authors gave it in [1,3-5,8,10-12], we find the circular retail for the groups $\mathcal{SL}(2, \mathfrak{U})$ where $\mathfrak{U} = 41$ and 43 .

2- Elementary Concepts

In this section some facts were mentioned.

Theorem 2.1: [6]

If G is any cyclic P -group, then $K(G) = Z_p$.

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Theorem 2.2: [6]

If G is any cyclic group of order Pn , then $K(G) = \bigoplus_{i=1}^n Z_{P^i}$.

Theorem 2.3: [2]

$$|\mathcal{SL}(2, p^k)| = p^k (p^{2k} - 1).$$

3. Primary account

We using the notion in [1,3-5,8,10-12] to find the circular retail for $\mathcal{SL}(2, \mathfrak{U})$ where $\mathfrak{U} = 41$ and 43.

3.1 The account for $\mathcal{SL}(2, 41)$

$$|\mathcal{SL}(2, 41)| = 68880.$$

The (ch.t.) of (r.rep.) for $\mathcal{SL}(2, 41)$ is

C_{π}	1	z	c	zc	a	a^2	a^4	b	b^2	b^3	b^6
$ C_{\pi} $	1	1	840	840	1722	1722	1722	1640	1640	1640	1640
$ C_G(g) $	68880	68880	82	82	40	40	40	42	42	42	42
1_G	1	1	1	1	1	1	1	1	1	1	1
Ψ	41	41	0	0	1	1	1	-1	-1	-1	-1
$\chi_1 + \chi_3 + \chi_5 + \chi_7 + \chi_9 + \chi_{11} + \chi_{13} + \chi_{15} + \chi_{17} + \chi_{19}$	420	-420	10	-10	0	0	0	0	0	0	0
$\chi_2 + \chi_4 + \chi_6 + \chi_8 + \chi_{12} + \chi_{14} + \chi_{16} + \chi_{18}$	336	336	8	8	0	0	-4	0	0	0	0
χ_{10}	42	42	1	1	0	-2	2	0	0	0	0
$\theta_1 + \theta_3 + \theta_5 + \theta_9 + \theta_{11} + \theta_{13} + \theta_{15} + \theta_{17} + \theta_{19}$	320	-320	-8	8	0	0	0	0	0	-3	3
θ_7	40	-40	-1	1	0	0	0	0	0	3	-3
$\theta_2 + \theta_4 + \theta_6 + \theta_8 + \theta_{10} + \theta_{12} + \theta_{16} + \theta_{18} + \theta_{20}$	360	360	-9	-9	0	0	0	-1	1	2	2
θ_{14}	40	40	-1	-1	0	0	0	1	1	-2	-2
$\xi_1 + \xi_2$	42	42	1	1	-2	2	2	0	0	0	0
$\eta_1 + \eta_2$	-40	-40	-1	-1	0	0	0	2	-2	2	-2

The diagonalization matrix of it is

$$\begin{pmatrix} 68880 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -17220 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 & 0 \end{pmatrix}$$

Thus

$$K(\mathcal{SL}(2, 41)) = Z_{68880} \oplus Z_{17220} \oplus Z_4 \oplus Z_2 \oplus Z_5 \oplus Z_4 \oplus Z_7 \oplus Z_2 \oplus Z_6 \oplus Z_1 \oplus Z_7$$

3.2 The account for $\mathcal{SL}(2,43)$

$$|\mathcal{SL}(2,43)| = 79464.$$

The (ch.t.) of (r.rep.) for $\mathcal{SL}(2,43)$ is

C_g	1	z	c	zc	a	a^2	a^3	a^6	b	b^2	b^4
$ C_g $	1	1	924	924	1892	1892	1892	1892	1806	1806	1806
$ C_G(g) $	79464	79464	86	86	42	42	42	42	44	44	44
1_G	1	1	1	1	1	1	1	1	1	1	1
Ψ	43	43	0	0	1	1	1	1	-1	-1	-1
$\chi_1 + \chi_3 + \chi_5 + \chi_7 + \chi_9 + \chi_{11} + \chi_{13} + \chi_{15} + \chi_{17} + \chi_{19}$	396	-396	9	-9	0	0	3	-3	0	0	0
χ_7	44	-44	1	-1	1	-1	-2	2	0	0	0
$\chi_2 + \chi_4 + \chi_6 + \chi_8 + \chi_{10} + \chi_{12} + \chi_{16} + \chi_{18} + \chi_{20}$	396	396	9	9	0	0	-3	-3	0	0	0
χ_{14}	44	44	1	1	-1	-1	2	2	0	0	0
$\theta_1 + \theta_3 + \theta_5 + \theta_9 + \theta_{11} + \theta_{13} + \theta_{15} + \theta_{17} + \theta_{19} + \theta_{21}$	420	-420	-10	10	0	0	0	0	0	-2	2
θ_{11}	42	-42	-1	1	0	0	0	0	0	2	-2
$\theta_2 + \theta_4 + \theta_6 + \theta_8 + \theta_{10} + \theta_{12} + \theta_{16} + \theta_{18} + \theta_{20}$	420	420	-10	-10	0	0	0	0	0	2	2
$\xi_1 + \xi_2$	44	-44	1	-1	-2	2	-2	2	0	0	0
$\eta_1 + \eta_2$	42	42	-1	-1	0	0	0	0	2	-2	-2

The diagonalization matrix of it is

$$\begin{pmatrix} 79464 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -19866 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix}$$

Thus

$$K(\mathcal{SL}(2,43)) = Z_{79464} \oplus Z_{19866} \oplus Z_2 \oplus Z_7 \oplus Z_2 \oplus Z_4 \oplus Z_2 \oplus Z_{11} \oplus Z_1 \oplus Z_3 \oplus Z_2$$

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